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## Mathematics

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## Introduction

Mathematics is one of six key learning areas for the primary curriculum. The Education Act 1990 (NSW) sets out minimum curriculum requirements for primary schools. It requires that courses of study must be provided in each of the key learning areas for primary education for each child during each year. This Mathematics K-6 Syllabus provides information about teaching and learning in Mathematics. It replaces the existing syllabus, Mathematics K-6 (1989) and the Mathematics K-6 Outcomes and Indicators (1998) document.

The Mathematics K-6 Syllabus is organised into six strands - one process strand, Working Mathematically, and the five content strands, Number, Patterns and Algebra, Data, Measurement, and Space and Geometry. Working Mathematically encompasses a set of five key processes that are embedded into the other five strands through the content. This relationship is represented in the following diagram. To aid further organisation, each of the five content strands has a set of substrands as indicated below.


The Mathematics $K-6$ Syllabus forms part of the continuum of mathematics learning from Kindergarten to Year 10. To ensure coherence and continuity, this syllabus was developed at the same time as the Mathematics Years 7-10 Syllabus. These syllabuses contain a common rationale, aim and objectives. In addition, the outcomes and content are organised into the same six strands. A K-10 Mathematics Scope and Continuum that describes the key ideas to be developed at each Stage, and for each strand, is also contained in both syllabuses.

The content presented in any particular Stage represents the knowledge, skills and understanding that are to be achieved by a typical student by the end of that Stage. It needs to be acknowledged that students learn at different rates and in different ways, so that there will be students who have not achieved the outcomes for the Stage/s prior to that identified with their stage of schooling. Teachers will need to identify these students and to plan learning experiences that provide opportunities to develop understanding of earlier concepts. In addition, there will be students who achieve the outcomes for their Stage before the end of their stage of schooling. These students will need learning experiences that develop understanding of concepts in the next Stage. In this way, students can move through the continuum at a faster rate. In order to cater for the full range of primary school students, Stage 4 outcomes and content have been included in this syllabus.

The syllabus is based on the recognition that students' formative learning experiences will often involve information technology. It acknowledges the increasing availability of computers in schools and in the home. It recognises the opportunities that students will have to acquire, interpret and create information by using computers and other technologies. Information technology enables students to locate, access, view and analyse a range of source material. In addition, it provides opportunities for students to design and create information products, and to determine the usefulness, accuracy, reliability and validity of information.

## Students with Special Education Needs

In $\mathrm{K}-6$, the syllabus provides for students with special education needs in a variety of ways:

- through the inclusion of outcomes and content which provide for the full range of students
- through the development of additional advice and programming support for teachers to assist students to access the outcomes of the syllabus
- through the development of specific support documents for students with special education needs.

In $\mathrm{K}-6$, teachers and parents plan together to ensure that syllabus outcomes and content reflect the learning needs and priorities of individual students.

It is necessary to focus on the individual needs, interests and abilities of each student when planning a program that will comprise the most appropriate combination of outcomes and content available.

## The K-10 Curriculum

This syllabus has been developed within the parameters set by the Board of Studies NSW in its K-10 Curriculum Framework. This framework ensures that K-10 syllabuses and curriculum requirements are designed to provide educational opportunities that:

- engage and challenge all students to maximise their individual talents and capabilities for lifelong learning
- enable all students to develop positive self-concepts, and their capacity to establish and maintain safe, healthy and rewarding lives
- prepare all students for effective and responsible participation in their society, taking account of moral, ethical and spiritual considerations
- encourage and enable all students to enjoy learning, and to be self-motivated, reflective, competent learners who will be able to take part in further study, work or training
- promote a fair and just society that values diversity
- promote continuity and coherence of learning and facilitate transition between primary and secondary schooling.
The framework also provides a set of broad learning outcomes that summarise the knowledge, skills and understanding, values and attitudes essential for all students to succeed in and beyond their schooling. These broad learning outcomes indicate that students will:
- understand, develop and communicate ideas and information
- access, analyse, evaluate and use information from a variety of sources
- work collaboratively with others to achieve individual and collective goals
- possess the knowledge and skills necessary to maintain a safe and healthy lifestyle
- understand and appreciate the physical, biological and technological world and make responsible and informed decisions in relation to their world
- understand and appreciate social, cultural, geographical and historical contexts and participate as active and informed citizens
- express themselves through creative activity and engage with the artistic, cultural and intellectual work of others
- understand and apply a variety of analytical and creative techniques to solve problems
- understand, interpret and apply concepts related to numerical and spatial patterns, structures and relationships
- be productive, creative and confident in the use of technology and understand the impact of technology on society
- understand the work environment and be equipped with the knowledge, skills and understanding to evaluate potential career options and pathways
- develop a system of personal values based on their understanding of moral, ethical and spiritual matters.

The way in which learning in the Mathematics K-6 Syllabus contributes to curriculum and to the student's achievement of the broad learning outcomes is outlined in the syllabus rationale.
In accordance with the $K-10$ Curriculum Framework, the Mathematics $K-6$ Syllabus takes into account the diverse needs of all students. It identifies essential knowledge, skills and understanding, values and attitudes. It enunciates clear standards of what students are expected to know and be able to do in K-6. It provides structures and processes by which teachers can provide continuity of study for all students, ensuring successful transition at all Stages from Kindergarten to Year 10.

The syllabus also assists students to maximise their achievement in mathematics through the acquisition of additional knowledge, skills and understanding, values and attitudes. It contains advice to assist teachers to program learning for those students who have gone beyond achieving the outcomes through their study of the essential content.

## Rationale for Mathematics in K-10

Mathematics is a reasoning and creative activity employing abstraction and generalisation to identify, describe and apply patterns and relationships. It is a significant part of the cultural heritage of many diverse societies. The symbolic nature of mathematics provides a powerful, precise and concise means of communication. Mathematics incorporates the processes of questioning, reflecting, reasoning and proof. It is a powerful tool for solving familiar and unfamiliar problems both within and beyond mathematics. As such, it is integral to scientific and technological advances in many fields of endeavour. In addition to its practical applications, the study of mathematics is a valuable pursuit in its own right, providing opportunities for originality, challenge and leisure.
The study of mathematics provides opportunities for students to learn to describe and apply patterns and relationships; reason, predict and solve problems; calculate accurately both mentally and in written form; estimate and measure; and interpret and communicate information presented in numerical, geometrical, graphical, statistical and algebraic forms. Mathematics in $\mathrm{K}-10$ provides support for concurrent learning in other key learning areas and builds a sound foundation for further mathematics education.

Students will have the opportunity to develop an appreciation of mathematics and its applications in their everyday lives and in the worlds of science, technology, commerce, the arts and employment. The study of the subject enables students to develop a positive self-concept as learners of mathematics, obtain enjoyment from mathematics, and become self-motivated learners through inquiry and active participation in challenging and engaging experiences.

The ability to make informed decisions, and to interpret and apply mathematics in a variety of contexts, is an essential component of students' preparation for life in the twenty-first century. To participate fully in society students need to develop the capacity to critically evaluate ideas and arguments that involve mathematical concepts or that are presented in mathematical form.

## Aim

The aim of Mathematics in $\mathrm{K}-10$ is to develop students' mathematical thinking, understanding, competence and confidence in the application of mathematics, their creativity, enjoyment and appreciation of the subject, and their engagement in lifelong learning.

## Objectives

## Knowledge, Skills and Understanding

Students will develop knowledge, skills and understanding:

- through inquiry, application of problem-solving strategies including the selection and use of appropriate technology, communication, reasoning and reflection
- in mental and written computation and numerical reasoning
- in patterning, generalisation and algebraic reasoning
- in collecting, representing, analysing and evaluating information
- in identifying and quantifying the attributes of shapes and objects and applying measurement strategies
- in spatial visualisation and geometric reasoning.


## Values and Attitudes

Students will:

- appreciate mathematics as an essential and relevant part of life
- show interest and enjoyment in inquiry and the pursuit of mathematical knowledge, skills and understanding
- demonstrate confidence in applying mathematical knowledge, skills and understanding to everyday situations and the solution of everyday problems
- develop and demonstrate perseverance in undertaking mathematical challenges
- recognise that mathematics has been developed in many cultures in response to human needs.


## Overview of Learning in Mathematics

This syllabus contains essential and additional content. The essential content is presented as outcomes and content statements in six strands. The additional content consists of non-mandatory topics that teachers may use to further broaden and enrich students' learning in mathematics. As well as the essential and additional content, particular cross-curriculum areas are incorporated into the content of the syllabus.

## Essential Content

The essential content for mathematics in $\mathrm{K}-10$ is structured using one process strand

- Working Mathematically,
and five content strands
- Number
- Patterns and Algebra
- Data
- Measurement
- Space and Geometry.

These strands contain the knowledge, skills and understanding for the study of mathematics in the compulsory years of schooling.

Strands are used as organisers of outcomes and content to assist teachers with planning, programming, assessment and reporting. From Early Stage 1 to Stage 3, the five content strands are organised into substrands and in Stage 4, the strands are organised into topics, as follows.

## Strand (and associated Objective)

## Early Stage 1 to Stage 3

Substrands

Stage 4 Topics

Working Mathematically
Students will develop knowledge, skills and understanding through inquiry, application of problem-solving strategies including the selection and use of appropriate technology, communication, reasoning and reflection.

## Five Interrelated Processes

Questioning
Applying Strategies
Communicating
Reasoning
Reflecting

| Number | Whole Numbers | Operations with Whole Numbers |
| :--- | :--- | :--- |
| Students will develop knowledge, skills and |  |  |
| understanding in mental and written computation |  |  |
| and numerical reasoning. | Addition and Subtraction <br> Multiplication and Division <br> Fractions and Decimals <br> Chance | Integers <br> Fractions, Decimals and Percentages <br> Probability |
| Patterns and Algebra Patterns and Algebra |  |  |
| Students will develop knowledge, skills and <br> understanding in patterning, generalisation and <br> algebraic reasoning. |  | Number Patterns |
| Algebraic Techniques |  |  |

Data D

Students will develop knowledge, skills and

Data Representation
Data Analysis and Evaluation
understanding in collecting, representing, analysing and evaluating information.

| Measurement | Length | Perimeter and Area |
| :--- | :--- | :--- |
| Students will develop knowledge, skills and <br> understanding in identifying and quantifying the <br> attributes of shapes and objects and applying <br> measurement strategies. | Area <br> Volume and Capacity | Mass <br> Surface Area and Volume |
| Time |  |  |
| Space and Geometry | Three-dimensional Space | Properties of Solids |
| Students will develop knowledge, skills and <br> understanding in spatial visualisation and | Two-dimensional Space | Position |

In each of the strands, particular aspects of students' mathematical learning and understanding are developed. However, students need to be able to make connections between mathematical ideas and concepts in order to develop a richer understanding and better appreciation of mathematics. Integrating concepts within and between the following strands will support development of these connections.

## Working Mathematically

Working Mathematically encompasses five interrelated processes. These processes come into play when developing new skills and concepts and also when applying existing knowledge to solve routine and non-routine problems both within and beyond mathematics. At times the focus may be on a particular process or group of processes, but often the five processes overlap. While this strand has a set of separate outcomes, it is integrated into the content of each of the five content strands in the syllabus.

## Number

Number encompasses the development of number sense and confidence and competence in using mental, written and calculator techniques for solving problems. Formal written algorithms are introduced after students have gained a firm understanding of basic concepts including place value, and have developed mental strategies for computing with two-digit and three-digit numbers.

## Patterns and Algebra

Patterns and Algebra has been incorporated into the primary curriculum to demonstrate the importance of early number learning in the development of algebraic thinking. This strand emphasises number patterns and number relationships leading to an investigation of the way that one quantity changes relative to another.

## Data

Data addresses the need for all students to understand, interpret and analyse information displayed in tabular and graphical forms. Students learn to ask questions relevant to their experiences and interests and to design ways of investigating their questions. They need to recognise when information has been displayed in a misleading manner that can result in false conclusions.

## Measurement

Measurement enables the identification and quantification of attributes of objects so that they can be compared and ordered. In this strand, each attribute is developed by the identification of the attribute and comparison of objects, the use of informal units, the use of formal units, as well as consideration of applications and generalisations. Students need to be able to select and use appropriate units and measuring tools, and to calculate areas and volumes given particular information.

## Space and Geometry

Space and Geometry is the study of spatial forms. It involves representation of shape, size, pattern, position and movement of objects in the three-dimensional world, or in the mind of the learner. Students learn to recognise, visualise and draw shapes and describe the features and properties of three-dimensional objects and two-dimensional shapes in static and dynamic situations.

## Additional Content

In addition to the essential content that relates to the outcomes listed in each of the strands, teachers may wish to include in their teaching and learning programs other material in order to broaden and deepen students' knowledge, skills and understanding, to meet students' interests, or to stimulate student interest in other areas of mathematics.

The following list contains possible topics for inclusion as Additional Content in teaching and learning programs. This additional content is not essential, nor is it required as prerequisite knowledge for other topics in the K-12 Mathematics curriculum. The list is not exhaustive.

## Number

Exploration of numbers such as perfect and amicable numbers
Venn diagrams
Number bases other than 10
Other calculating methods eg Peasant method, Egyptian method
Other calculating devices eg abacus, Napier's Bones
Other monetary systems
Construction of magic squares
Logic puzzles
Number theory
Codes

## Measurement

The history of the calendar
The history of other measuring devices such as sundials
History of measurement in Australia
Other measurement systems - when studying another culture in Human Society and its Environment (HSIE)
Temperature - use of various thermometers and temperature scales
Unusual units of measurement
Navigation - latitude and longitude in relation to HSIE units

## Space and Geometry

Knots
Further tessellations (including semi-regular tessellations)
Semi-regular polyhedra; truncated, snub-nosed and stellated solids

## Cross-curriculum Content

The Board of Studies has developed cross-curriculum content that is to be included in the outcomes and content of syllabuses. The identified content will be incorporated appropriately in $\mathrm{K}-10$ syllabuses. The cross-curriculum content addresses issues, perspectives and policies that will assist students to achieve the broad learning outcomes defined in the Board of Studies $K-10$ Curriculum Framework. The cross-curriculum content statements have been developed in accordance with the requirement of the K-10 Curriculum Framework that 'syllabuses will include cross-curriculum content that is appropriate to teach in the key learning area or subject'.

The statements act as a mechanism to embed cross-curriculum content into all syllabuses for $\mathrm{K}-10$. Knowledge, skills, understanding, values and attitudes derived from the cross-curriculum content areas will be included in Board syllabuses, while ensuring that subject integrity is maintained.

## Information and Communication Technology

Information and Communication Technology (ICT) has been developed with the significant utilisation of mathematics, and a range of opportunities exists within the teaching and learning of mathematics to utilise ICT. For example, spreadsheets can be used to record, organise and manipulate numbers in Number, Patterns and Algebra, and Data. Basic draw and paint programs can be used to create shapes and designs in Space and Geometry and repeating patterns in Patterns and Algebra. Problem-solving software can be used to explore problems relevant to all strands.

## Work, Employment and Enterprise

Work, Employment and Enterprise content enables students to develop work-related knowledge, skills and understanding through their study of mathematics. It also provides opportunities for students to develop values and attitudes about work, employment and the workplace.

Specifically this occurs through student study of mathematics in work-related contexts, through selecting and applying appropriate mathematical techniques and problem-solving strategies, and in acquiring, processing, assessing and communicating information.

## Numeracy

Numeracy is the ability to effectively use the mathematics required to meet the general demands of life at home and at work, and for participation in community and civic life. As a field of study, mathematics is developed and/or applied in situations that extend beyond the general demands of everyday life.
Numeracy is a fundamental component of learning across all areas of the curriculum. The development and enhancement of students' numeracy skills and understanding is the responsibility of teachers across different learning areas that make specific demands on student numeracy.

To be numerate is to use mathematical ideas effectively to make sense of the world. Numeracy involves drawing on knowledge of particular contexts and circumstances in deciding when to use mathematics, choosing the mathematics to use, and critically evaluating its use. Numeracy incorporates the disposition to use numerical, spatial, graphical, statistical and algebraic concepts and skills in a variety of contexts and involves the critical evaluation, interpretation, application and communication of mathematical information in a range of practical situations.

The key role that teachers of mathematics play in the development of numeracy includes teaching students specific skills and providing them with opportunities to select, use, evaluate and communicate mathematical ideas in a range of situations. Students' numeracy and underlying mathematical understanding will be enhanced through engagement with a variety of applications of mathematics to real-world situations and problems in other key learning areas.

## Key Competencies

Key Competencies are generic competencies essential for effective participation in existing and emerging learning for future education, work and life in general. The Mathematics K-6 Syllabus provides a powerful context within which to develop general competencies considered essential for the continuing development of those effective thinking skills which are necessary for further education, work and everyday life. The knowledge, skills and understanding that underpin the key competencies are taught by making them explicit, designing learning tasks that provide opportunities to develop them, and identifying specific criteria for their assessment.

Key competencies are embedded in the Mathematics K-6 Syllabus to enhance student learning. They are incorporated into the objectives, outcomes and content of the syllabus and/or are developed through classroom teaching. The key competencies are:

- collecting, analysing and organising information
- communicating ideas and information
- planning and organising activities
- working with others and in teams
- using mathematical ideas and techniques
- solving problems
- using technology.

This syllabus explicitly addresses knowledge and skills that provide students with opportunities to collect, analyse and organise information numerically and graphically.

Mathematics contributes to the development of students' abilities to communicate ideas and information by facilitating the development of skills in interpreting and representing information in numerical, algebraic, statistical and graphical forms. Students are encouraged to express mathematical concepts and processes using their own words as well as using mathematical terminology and notation.
Problem-solving tasks provide opportunities for students to develop the capacity to plan and organise activities. Planning and organising their own strategies for obtaining solutions to tasks involves the ability to set goals, establish priorities, implement a plan, select and manage resources and time, and monitor individual performance.

The experience of working with others and in teams can facilitate learning. Groupwork provides the opportunity for students to communicate mathematically with each other, to make conjectures, to cooperate and to persevere when solving problems and undertaking investigations.

Throughout the syllabus, students are developing the key competencies using mathematical ideas and techniques and solving problems. Across the syllabus strands attention is drawn to opportunities for students to solve meaningful and challenging problems in both familiar and unfamiliar contexts, within mathematics, in other key learning areas, at work and in everyday situations. Problem solving can promote communication, critical reflection, creativity, analysis, organisation, experimentation, synthesis, generalisation, validation, perseverance, and systematic recording of information. In addition, teaching through problems that are relevant to students can encourage improved attitudes to mathematics and an appreciation of its importance to society.
In order to achieve the outcomes of this syllabus, students will need to learn about and use appropriate technologies to develop the key competency using technology. It is important for students to determine the purpose of a technology, when and how to apply the technology, and to evaluate the effectiveness of its application, or whether its use is inappropriate or even counterproductive. Computer software as well as calculators can be used to facilitate teaching and learning.

## Literacy

Literacy is the ability to communicate purposefully and appropriately with others, in and through a wide variety of contexts, modes and mediums. While English has a particular role in developing literacy, all curriculum areas, including mathematics, have a responsibility for the general literacy requirements of students, as well as for the literacy demands of their particular discipline.

Studies have shown that the causes of student errors on word problems may relate to the literacy components rather than the application of mathematical computations. Mathematics at times uses words from everyday language that have different meanings within a mathematical context. This can create confusion for some students. Clear explanations of these differences will assist students in the acquisition and use of mathematical terminology.
The growth of technology and information, including visual information, demands that students be critically, visually and technologically literate and can compose, acquire, process, and evaluate text in a wide variety of contexts. They need to understand the full scope of a text's meaning, including the wide contextual factors that take meaning beyond a decoding process.


## Foundation Statements

Foundation Statements set out a clear picture of the knowledge, skills and understanding that each student should develop at each stage of primary school.

## Prior-to-school Learning

Teachers need to acknowledge the learning that children bring to school, and plan appropriate learning experiences that make connections with existing mathematical understanding. Children start developing mathematical understanding well before they start school since mathematics is a part of everyday life. In addition, many children will have participated in playgroup, childcare or pre-school programs.
As children engage in daily life they construct mathematical understanding that is often enhanced by planned mathematical experiences in prior-to-school settings. Such understanding may include the development of number recognition, number representation and oral counting sequences, spatial awareness and shape recognition. In addition, vocabulary development is evident as students begin to acquire everyday language associated with length, area, volume, mass, time and position. Teachers need to become familiar with children's existing mathematical understanding as they commence school to ensure that programming is designed to meet the needs of individual students.
Early Stage 1 outcomes may not be the most appropriate starting point for all students. For some students, it will be appropriate to focus on these outcomes whereas others will benefit from a focus on more basic mathematical concepts. Still others may demonstrate understanding beyond Early Stage 1. The movement into Early Stage 1 should be seen as a continuum of mathematical learning. To ensure this continuum is maintained, teachers need to base their planning on the evaluation of current understanding related to all of the strands.

## Early Stage 1

## Working Mathematically $\square$ Number $\square$ Patterns and Algebra ■ Measurement and Data ■ Space and Geometry

Students ask questions and explore mathematical problems. They use everyday language, materials and informal recordings to demonstrate understanding and link mathematical ideas. Students count to 30 and represent numbers to 20 with objects, pictures, numerals and words and read and use ordinal numbers to at least 'tenth' place. They manipulate objects to model addition and subtraction, multiplication and division. Students divide objects into two equal parts and describe them as halves. They recognise coins and notes. Students recognise, describe and continue patterns that increase or decrease.
Students identify length, area, volume, capacity and mass and compare and arrange objects according to these attributes.
They name the days of the week and the seasons and they order events in a school day, telling the time on the hour. Students use objects and pictures to create a data display and interpret data.
Students manipulate, sort and describe 3D objects using everyday language. They manipulate, sort and describe 2D shapes, identifying circles, squares, triangles and rectangles. Students give and follow simple directions and describe position using everyday language.

Stage 1
Working Mathematically $\square$ Number $\square$ Patterns and Algebra ■ Measurement and Data $\square$ Space and Geometry
Students ask questions and use objects, diagrams and technology to explore mathematical problems. They link mathematical ideas and use everyday language, some mathematical language and diagrams to explain how answers were obtained.
Students count, order, read and write numbers up to 999 and use a range of mental strategies, informal recording methods and materials to add, subtract, multiply and divide. They model and describe objects and collections divided into halves and quarters. Students sort, order and count money and recognise and describe the element of chance in familiar activities.
Students describe, create and continue a variety of number patterns and relate addition and subtraction facts to at least 20.
Students estimate, measure, compare and record using informal units for length, area, volume, capacity and mass. They recognise the need for formal units of length and use the metre and centimetre to measure length and distance. Students use a calendar to identify the date and name and order the months and the seasons of the year. They use informal units to compare and order the duration of events and tell the time on the half-hour. Students gather, organise, display and interpret data using column and picture graphs. Students identify, describe, sort and model particular 3D objects and 2D shapes. They represent and describe the position of objects.

## Stage 2

## Working Mathematically $\square$ Number $\square$ Patterns and Algebra $\square$ Measurement and Data $\square$ Space and Geometry

Students ask questions and use appropriate mental or written strategies, and technology, to solve problems. They use appropriate terminology to describe and link mathematical ideas, check statements for accuracy and explain reasoning. Students count, order, read and record numbers up to 9999 and use mental and written strategies, including the formal written algorithm, to solve addition and subtraction problems involving numbers of up to four digits. They use mental strategies to recall multiplication facts up to $10 \times 10$ and related division facts and use informal written strategies for multiplication and division of two-digit numbers by one-digit numbers. Students model, compare and represent simple fractions and recognise percentages in everyday situations and they model, compare, represent, add and subtract decimals to two decimal places. Students perform simple calculations with money and conduct simple chance experiments.
Students generate, describe and record number patterns and relate multiplication and division facts to at least $10 \times 10$.
Students estimate, measure, compare and record length, area, volume, capacity and mass using some formal units. They read and record time in hours and minutes in digital and analogue notation and make comparisons between time units. Students gather and organise data to create and interpret tables and graphs.
Students name, describe and sketch particular 3D objects and 2D shapes. They compare angles using informal means and describe a 'right angle'. Students use coordinates to describe position and compass points to give and follow directions.

## Stage 3

## Working Mathematically $\square$ Number $\square$ Patterns and Algebra $■$ Measurement and Data $\square$ Space and Geometry

Students ask questions and undertake investigations, selecting appropriate technological applications and problem-solving strategies. They use mathematical terminology and some conventions and they give valid reasons when comparing and selecting from possible solutions, making connections with existing knowledge and understanding.
Students read, write and order numbers of any size, selecting and applying appropriate mental, written or calculator strategies for the four operations. They compare, order and perform calculations with simple fractions, decimals and simple percentages and apply the four operations to money in real-life situations. Students place the likelihood of simple events in order on a number line from 0 to 1 .
Students record and describe geometric and number patterns using tables and words. They construct, verify and complete number sentences involving the four operations.
Students select and use the appropriate unit to estimate, measure and calculate length, area, volume, capacity and mass. They use 24 -hour time in real-life situations and construct timelines. Students draw and interpret a variety of graphs using a scale
Students construct and classify 3D objects and 2D shapes and compare and describe their properties. They measure, construct and classify angles and make simple calculations using scale.

## Stage 4

Students who have achieved Stage 4 outcomes use mathematical terminology, algebraic notation, diagrams, text and tables to communicate mathematical ideas, and link concepts and processes within and between mathematical contexts. They apply their mathematical skills and understanding in analysing real-life situations and in systematically formulating questions or problems that they then explore and solve, using technology where appropriate. In solving particular problems, they compare the strengths and weaknesses of different strategies and solutions.
Students have developed a range of mental strategies to enhance their computational skills. They operate competently with directed numbers, fractions, percentages, mixed numerals and decimals and apply these in a range of practical contexts, including problems related to discounts and profit and loss. They are familiar with the concepts of ratio, rates and the probability of simple and complementary events and apply these when solving problems. They use index notation for numbers with positive integral indices and explore prime factorisation, squares and related square roots, and cubes and related cube roots. Students investigate special groups of positive whole numbers, divisibility tests and other counting systems.
Extending and generalising number patterns leads students into an understanding of the use of pronumerals and the language of algebra, including the use of index notation. Students simplify algebraic expressions, substitute into algebraic expressions and formulae, and expand and factorise algebraic expressions. They solve simple linear equations, inequalities, and word problems. They develop tables of values from simple relationships and illustrate these relationships on the number plane.
Students construct and interpret line, sector, travel, step and conversion graphs, dot plots, stem-and-leaf plots, divided bar graphs, and frequency tables and histograms. In analysing data, they consider both discrete and continuous variables, sampling versus census, prediction and possible misrepresentation of data, and calculate the mean, mode, median and range.
Students find the area and perimeter of a variety of polygons, circles, and simple composite figures, the surface area and volume of rectangular and triangular prisms, and the volume of cylinders and right prisms. Pythagoras' theorem is used to calculate the distance between two points. They describe the limit of accuracy of their measures, interpret and use tables and charts related to time, and apply their understanding of Australian and world time zones to solve problems.
Their knowledge of the properties of two- and threedimensional geometrical figures, angles, parallel lines, perpendicular lines, congruent figures, similar figures and scale drawings enables them to solve numerical exercises on finding unknown lengths and angles in figures.


## Overview of Outcomes

Syllabus outcomes are specific statements of the results intended by the syllabus. These outcomes are achieved as students engage with the content of the syllabus. They are arranged in strands that follow a conceptual sequence from Early Stage 1 through to Stage 4. The outcomes are statements of the knowledge, skills and understanding to be achieved by most students as a result of effective teaching and learning of mathematics by the end of each Stage. For example, by the end of Year 6, it is expected that most students are able to demonstrate achievement of Stage 3 outcomes to some level.

Learning however occurs at different rates and in different ways. Therefore, there will be variability in the achievement of Stage outcomes during particular Years of schooling. For example, in Year 6 there are some students who have learning needs that will determine that they should be working towards outcomes at an earlier Stage or at a later Stage.

For students who have achieved and are working beyond Stage 3 during the primary Years, Stage 4 has been included.
A code has been applied to each of the outcomes to facilitate reference throughout the syllabus.

| WM | Working Mathematically |
| :--- | :--- |
| N | Number |
| PA | Patterns and Algebra |
| D | Data |
| M | Measurement |
| SG | Space and Geometry |

For example, the following outcome:
NS2.3 Uses mental and informal written strategies for multiplication and division
refers to an outcome from the Number strand in Stage 2. The last number indicates that this outcome belongs to the third set of Number outcomes.

In the Stages from Early Stage 1 to Stage 3, where there is more than one outcome for a substrand at a particular Stage, the code ends with 'a' or 'b' to indicate the first or second outcome.

For example, the following two outcomes are included in Two-dimensional Space for Stage 3:
SGS3.2a Manipulates, classifies and draws two-dimensional shapes and describes side and angle properties

SGS3.2b Measures, constructs and classifies angles

## Working Mathematically Outcomes and Indicators

There is no specific list of knowledge and skills for the Working Mathematically strand. The Working Mathematically processes have been embedded in the content section of this syllabus and appear on each of the content pages. The set of indicators for each of the Working Mathematically outcomes will help teachers to assess this strand. It should be noted that this is not a comprehensive list. Teachers are encouraged to design their own indicators for the assessment of Working Mathematically.

The wording of the outcomes for Questioning and Reflecting is the same for each Stage except for the last part of the statement, which indicates that the outcome should be assessed in relation to the relevant content for that Stage. This is not to suggest that there is no development of these two processes across Stages. Development of these processes is closely linked to the development of the content and needs to be assessed in relation to the content. For Questioning, this means that a student working towards Early Stage 1 might ask a question about counting forwards or backwards, whereas a student working towards Stage 3 might ask a question about creating sixths of a collection of objects. For Reflecting, a student working towards Stage 1 might identify the use of numbers in the school and neighbourhood, whereas a student working towards Stage 4 might be able to identify the use of a variety of mathematical ideas in other cultures.

## Working Mathematically Outcomes

## Process

## Questioning

Students ask questions in relation to mathematical situations and their mathematical experiences

Applying Strategies
Students develop, select and use a range of strategies, including the selection and use of appropriate technology, to explore and solve problems

Communicating
Students develop and use appropriate language and representations to formulate and express mathematical ideas

## Reasoning

Students develop and use processes for exploring relationships, checking solutions and giving reasons to support their conclusions

## Reflecting

Students reflect on their experiences and critical understanding to make connections with, and generalisations about, existing knowledge and understanding

## EARLY STAGE 1

WMES1.1
Asks questions that could be explored using mathematics in relation to Early Stage 1 content

WMES1.2
Uses objects, actions, imagery, technology and/or trial and error to explore mathematical problems

WMES1.3
Describes mathematical situations using everyday language, actions, materials, and informal recordings

WMES1. 4
Uses concrete materials and/or pictorial representations to support conclusions

WMES1.5
Links mathematical ideas and makes connections with, and generalisations about, existing knowledge and understanding in relation to Early Stage 1 content

WMS1.3
Describes mathematical situations and methods using everyday and some mathematical language, actions, materials, diagrams and symbols

WMS1.4
Supports conclusions by explaining or demonstrating how answers were obtained
STAGE 1

WMS1.1
Asks questions that could be explored using mathematics in relation to Stage 1 content

WMS1. 2
Uses objects, diagrams, imagery and technology to explore mathematical problems都

WMS1.5
Links mathematical ideas and makes connections with, and generalisations about, existing knowledge and understanding in relation to Stage 1 content

STAGE 2

WMS2.1
Asks questions that could be explored using mathematics in relation to Stage 2 content

WMS2.2
Selects and uses appropriate mental or written strategies, or technology, to solve problems

WMS2.3
Uses appropriate terminology to describe, and symbols to represent, mathematical ideas

WMS2.4
Checks the accuracy of a statement and explains the reasoning used

WMS2.5
Links mathematical ideas and makes connections with, and generalisations about, existing knowledge and understanding in relation to Stage 2 content

## WMS3.3

Describes and represents a mathematical situation in a variety of ways using mathematical terminology and some conventions
STAGE 3

WMS3.1
Asks questions that could be explored using mathematics in relation to Stage 3 content

WMS3.2
Selects and applies appropriate problem-solving strategies, including technological applications, in undertaking investigations

WMS3.4
Gives a valid reason Identifies for supporting one relationships and possible solution the strengths and over another

WMS3.5
Links mathematical ideas and makes connections with, and generalisations about, existing knowledge and understanding in relation to Stage 3 content

STAGE 4

WMS4. 1
Asks questions that could be explored using mathematics in relation to Stage 4 content

WMS4.2
Analyses a mathematical or real-life situation, solving problems using technology where appropriate

WMS4. 3
Uses mathematical terminology and notation, algebraic symbols, diagrams, text and tables to communicate mathematical ideas weaknesses of different strategies and solutions, giving reasons

WMS4.5
Links mathematical ideas and makes connections with, and generalisations about, existing knowledge and understanding in relation to Stage 4 content

## Number Outcomes

## Substrand <br> Whole Numbers <br> Students develop a sense of the relative

 size of whole numbers and the role of place value in their representation
## Addition and Subtraction

Students develop facility with number facts and computation with progressively larger numbers in addition and subtraction and an appreciation of the relationship between those facts

## Multiplication and Division

Students develop facility with number facts and computation with progressively larger numbers in multiplication and division and an appreciation of the relationship between those facts

## Fractions and Decimals

Students develop an understanding of the parts of a whole, and the relationships between the different
representations of fractions

Chance
Students develop an No outcome at this understanding of Stage the application of chance in everyday situations and an appreciation of the difference between theoretical and experimental probabilities

## EARLY STAGE 1 STAGE 1

NES1.1
Counts to 30, and orders, reads and represents numbers in the range 0 to 20

NS1.1
Counts, orders, reads and represents two- and three-digit numbers

STAGE 2

NS2. 1
Counts, orders reads and records numbers up to four digits

## NS2. 2

Uses mental and written strategies for addition and subtraction involving two-, three- and four-digit numbers

## STAGE 3

NS3.1
Orders, reads and writes numbers of any size NS3.2
Selects and applies appropriate strategies for addition and subtraction with counting numbers of any size

## NS3. 3

Selects and applies appropriate strategies for multiplication and division

## NS2.5

Describes and compares chance events in social and experimental contexts

## NS3. 4

Compares, orders and calculates with decimals, simple
fractions and simple percentages

## NS1.5

Recognises and describes the element of chance in everyday events
occurring in everyday situations
NS1.4
Describes and models halves and quarters, of objects and collections,

## NS2.4

Models, compares and represents commonly used fractions and decimals, adds and subtracts decimals to two decimal places, and interprets everyday percentages

## NS2. 3

Uses mental and informal written strategies for multiplication and division

 ges
NS2.5
Describes and
compares chance
events in social and
experimental
contexts

NS3.5
Orders the
likelihood of simple events on a number line from zero to one

STAGE 4

## Operations with Whole Numbers

 NS4. 1Recognises the properties of special groups of whole numbers and applies a range of strategies to aid computation

## Integers

NS4.2
Compares, orders and calculates with integers

## Fractions,

 Decimals and Percentages NS4.3Operates with fractions, decimals, percentages, ratios and rates

## Probability

NS4.4
Solves probability problems involving simple events

## Patterns and Algebra Outcomes

## Substrand

## Patterns and

## Algebra

Students develop skills in creating, describing and recording number patterns as well as an understanding of the relationships between numbers

PAES1.1
Recognises, describes, creates and continues repeating patterns and number patterns that increase or decrease number relationships

PAS1.1
Creates, represents and continues a variety of number patterns, supplies missing elements in a pattern and builds

STAGE 2

PAS2.1
Generates, describes and records number patterns using a variety of strategies and completes simple number sentences by calculating missing values

STAGE 3
STAGE 4

Algebraic Techniques
PAS4.1
Uses letters to represent numbers and translates between words and algebraic symbols

PAS3.1a
Records, analyses and describes geometric and number patterns that involve one operation using tables and words PAS3.1b
Constructs, verifies and completes number sentences involving the four operations with a variety of numbers

## Data Outcomes

Substrand

## Data

Students inform their inquiries through gathering, organising, tabulating and graphing data

EARLY STAGE 1 STAGE 1
DES1.1
Represents and interprets data displays made from objects and pictures

## DS1.1 <br> Gathers and

 organises data, displays data using column and picture graphs, and interprets the results
## STAGE 2

DS2.1
Gathers and organises data, displays data using tables and graphs, and interprets the results

STAGE 3
DS3.1
Displays and interprets data in graphs with scales of many-to-one correspondence

STAGE 4

Data
Representation
DS4.1
Constructs, reads and interprets graphs, tables, charts and statistical information

Data Analysis and Evaluation
DS4.2
Collects statistical data using either a census or a sample, and analyses data using measures of location and range

## Measurement Outcomes

## Substrand

## Length

Students distinguish the attribute of length and use informal and metric units for measurement

EARLY STAGE 1 STAGE 1

MES1.1
Describes length and distance using everyday language and compares lengths using direct comparison

Area
Students distinguish the attribute of area and use informal and metric units for measurement

MS1.1
Estimates, informal units, metres and centimetres

MS1.2
Estimates, and records areas

MES1.2
Describes area using everyday language and compares areas using direct comparison

MES1.3
Compares the capacities of containers and the volumes of objects or substances using direct comparison measures, compares and records lengths and distances using measures, compares using informal units

## MS1.3

Estimates, measures, compares and records volumes and capacities using informal units

## MES1.4

Compares the masses of two objects and describes mass using everyday language

Time
Students develop an understanding of the passage of time, its measurement and representations, through the use of everyday language and experiences

MS1.4
Estimates, measures, compares and records the masses of two or more objects using informal units

MES1.5
Sequences events and uses everyday language to describe the duration of activities

MS1.5
Compares the duration of events using informal methods and reads clocks on the halfhour

MS2. 2
Estimates, measures, compares and records the areas of surfaces in square centimetres and square metres

## STAGE 2

## MS2.1

Estimates, measures, compares and records lengths, distances and perimeters in metres, centimetres and millimetres

## MS2. 3

Estimates,
measures, compares and records volumes and capacities using litres, millilitres and cubic centimetres

STAGE 3

MS3. 1
Selects and uses the appropriate unit and device to measure lengths, distances and perimeters

## MS3.2

Selects and uses the appropriate unit to calculate area, including the area of squares, rectangles and triangles

## MS3.3

Selects and uses the appropriate unit to estimate and measure volume and capacity, including the volume of rectangular prisms

## MS2.4

Estimates, measures, compares and records masses using kilograms and grams

## MS2.5

Reads and records time in one-minute intervals and makes comparisons between time units

MS3.4
Selects and uses the appropriate unit and measuring device to find the mass of objects

MS3.5
Uses twenty-four hour time and am and pm notation in real-life situations and constructs timelines

STAGE 4

## Perimeter and Area

MS4. 1
Uses formulae and Pythagoras' theorem in calculating perimeter and area of circles and figures composed of rectangles and triangles

## Surface Area and Volume MS4. 2

Calculates surface area of rectangular and triangular prisms and volume of right prisms and cylinders

## Time

MS4.3
Performs
calculations of time that involve mixed units

## Space and Geometry Outcomes

Substrand
Three-dimensional SGES1.1

## Space

Students develop verbal, visual and mental representations of three-dimensional objects, their parts and properties, and different orientations

## Two-dimensional Space

Students develop verbal, visual and mental representations of lines, angles and two-dimensional shapes, their parts and properties, and different orientations

## Position

Students develop their representation of position through precise language and the use of grids and compass directions

EARLY STAGE 1

Manipulates, sorts and represents three-dimensional objects and describes them using everyday language

SGES1.2
Manipulates, sorts and describes representations of two-dimensional shapes using everyday language

STAGE 1

SGS1.1
Sorts, describes and represents threedimensional objects including cones, cubes, cylinders, spheres and prisms, and recognises them in pictures and the environment

SGS1. 2
Manipulates, sorts, represents, describes and explores various two-dimensional shapes

SGS2.2b
Identifies, compares and describes angles in practical situations

STAGE 2

SGS2.1
Makes, compares, describes and names threedimensional objects including pyramids, and represents them in drawings

SGS2.2a
Manipulates, compares, sketches and names twodimensional shapes and describes their features都

SGS1. 3
Represents the position of objects using models and drawings and describes using everyday language

## SGES1.3

Uses everyday language to describe position and give and follow simple directions

STAGE 3

SGS3. 1
Identifies threedimensional objects, including particular prisms and pyramids, on the basis of their properties, and visualises, sketches and constructs them given drawings of different views

## SGS3.2a

Manipulates, classifies and draws two-dimensional shapes and describes side and angle properties

SGS3.2b
Measures, constructs and classifies angles

STAGE 4

Properties of Solids SGS4. 1 Describes and sketches threedimensional solids including polyhedra, and classifies them in terms of their properties

## Properties of Geometrical

 FiguresSGS4.3
Classifies, constructs, and determines the properties of triangles and quadrilaterals SGS4. 4
Identifies congruent and similar twodimensional figures stating the relevant conditions

## Angles

SGS4. 2
Identifies and names angles formed by the intersection of straight lines, including those related to transversals on sets of parallel lines, and makes use of the relationships between them

## SGS2. 3

Uses simple maps and grids to represent position and follow routes

SGS3. 3
Uses a variety of mapping skills


Mathematics K-6

This section of the syllabus contains the K-10 Mathematics Scope and Continuum, outlines the presentation of the content pages, presents additional information about the Working Mathematically strand, and details the content in each of the strands Number, Patterns and Algebra, Data, Measurement, and Space and Geometry.

## K-10 Mathematics Scope and Continuum

The K-10 Mathematics Scope and Continuum (pp 28-37) is an overview of Key Ideas in each of the strands: Number, Patterns and Algebra, Data, Measurement, and Space and Geometry. For Early Stage 1 to Stage 3, the Scope and Continuum is organised into strands and substrands. For Stages 4 and 5, the Scope and Continuum is organised into strands and topics. These key ideas are also included on every page of the essential content that follows the Scope and Continuum.

The concepts in each of these strands are developed across the Stages to show how understanding in the early years needs to precede understanding in later years. In this way, the Scope and Continuum provides an overview of the sequence of learning for particular concepts in mathematics and links content typically taught in primary mathematics classrooms with content that is typically taught in secondary mathematics classrooms. It illustrates assumptions about prior learning and indicates pathways for further learning.

The essential content presented in any particular Stage represents the knowledge, skills and understanding that are to be achieved by a typical student by the end of that Stage. It needs to be acknowledged that students learn at different rates and in different ways, so that there will be students who have not achieved the outcomes for the Stage/s prior to that identified with their chronological age. Teachers will need to identify these students and to plan learning experiences that provide opportunities to develop understanding of concepts.
Each Stage builds upon the knowledge, skills and understanding developed in earlier Stages. For each Stage only new material is recorded in the Scope and Continuum. That is, for example, the content of Stage 4 builds on and extends the mathematics introduced in the previous Stages.

Students may be at different Stages for different strands of the Scope and Continuum. For example, a student may be working on Stage 3 content in the Number strand but be working on Stage 2 content in the Space and Geometry strand.
It is not intended that the Scope and Continuum be used as a checklist of teaching ideas. Rather, a variety of learning experiences needs to be planned and presented to students to maximise opportunities for achievement of outcomes. Students need appropriate time to explore, experiment and engage with the underpinning concepts and principles of what they are to learn.

It should be noted that the Working Mathematically strand does not appear in the Scope and Continuum as it does not have content and key ideas. It is written as outcomes that are presented on page 19.

## Scope and Continuum of Key Ideas: Number

|  | Early Stage 1 | Stage 1 | Stage 2 | Stage 3 |
| :---: | :---: | :---: | :---: | :---: |
|  | Count forwards to 30 , from a given number <br> Count backwards from a given number, in the range 0 to 20 <br> Compare, order, read and represent numbers to at least 20 <br> Read and use the ordinal names to at least 'tenth' <br> Use the language of money | Count forwards and backwards by ones, twos and fives <br> Count forwards and backwards by tens, on and off the decade <br> Read, order and represent two- and three-digit numbers <br> Read and use the ordinal names to at least 'thirty-first' <br> Sort, order and count money using face value | Count forwards and backwards by tens or hundreds, on and off the decade <br> Use place value to read, represent and order numbers up to four digits <br> Money concepts are developed further in Fractions and Decimals | Identify differences between Roman and Hindu-Arabic counting systems <br> Read, write and order numbers of any size using place value Record numbers in expanded notation <br> Recognise the location of negative numbers in relation to zero Money concepts are developed further in Fractions and Decimals |
|  | Combine groups to model addition Take part of a group away to model subtraction <br> Compare groups to determine 'how many more' <br> Record addition and subtraction informally | Model addition and subtraction using concrete materials Develop a range of mental strategies and informal recording methods for addition and subtraction <br> Record number sentences using drawings, numerals, symbols and words | Use a range of mental strategies for addition and subtraction involving two-, three- and four-digit numbers Explain and record methods for adding and subtracting <br> Use a formal written algorithm for addition and subtraction | Select and apply appropriate mental, written or calculator strategies for addition and subtraction with counting numbers of any size |
|  | Model equal groups or rows <br> Group and share collections of objects equally <br> Record grouping and sharing informally | Rhythmic and skip count by ones, twos, fives and tens Model and use strategies for multiplication including arrays, equal groups and repeated addition Model and use strategies for division including sharing, arrays and repeated subtraction <br> Record using drawings, numerals, symbols and words | Develop mental facility for number facts up to $10 \times 10$ <br> Find multiples and squares of numbers <br> Interpret remainders in division problems <br> Determine factors for a given number <br> Use mental and informal written strategies for multiplying or dividing a two-digit number by a one-digit operator | Select and apply appropriate mental, written or calculator strategies for multiplication and division <br> Explore prime and composite numbers <br> Use formal written algorithms for multiplication (limit operators to two-digit numbers) and division (limit operators to single digits) |
|  | Divide an object into two equal parts <br> Recognise and describe halves <br> Early money concepts are developed in Whole Numbers | Model and describe a half or a quarter of a whole object Model and describe a half or a quarter of a collection of objects Use fraction notation $\frac{1}{2}$ and $\frac{1}{4}$ | Model, compare and represent fractions with denominators 2,4 , and 8 , followed by fractions with denominators 5,10 , and 100 <br> Find equivalence between halves, quarters and eighths; fifths and tenths; tenths and hundredths <br> Model, compare and represent decimals to 2 decimal places Add and subtract decimals with the same number of decimal places (to 2 decimal places) <br> Recognise percentages in everyday situations. Relate a common percentage to a fraction or decimal Perform calculations with money | Model, compare and represent commonly used fractions (those with denominators $2,3,4,5,6,8$, 10, 12 and 100) <br> Find equivalence between thirds, sixths and twelfths <br> Express a mixed numeral as an improper fraction, and vice versa <br> Add and subtract simple fractions where one denominator is a multiple of the other Multiply simple fractions by whole numbers. Calculate unit fractions of a number <br> Multiply and divide decimals by whole numbers in everyday contexts. Add and subtract decimals to three decimal places Calculate simple percentages of quantities <br> Apply the four operations to money in real-life situations |
| $\begin{aligned} & \text { U } \\ & \text { U } \\ & \text { تِ } \\ & \text { U } \end{aligned}$ |  | Recognise the element of chance in familiar daily activities <br> Use familiar language to describe the element of chance | Explore all possible outcomes in a simple chance situation <br> Conduct simple chance experiments <br> Collect data and compare likelihood of events in different contexts | Assign numerical values to the likelihood of simple events occurring <br> Order the likelihood of simple events on a number line from 0 to 1 |

Scope and Continuum of Key Ideas: Number

| Stage 4 | Stage 5.1 | Stage 5.2 | Stage 5.3 |
| :--- | :--- | :--- | :--- |

## Operations with Whole

## Numbers

Explore other counting systems
Investigate groups of positive
whole numbers
Apply mental strategies to aid computation

## Integers

Perform operations with directed numbers
Simplify expressions involving grouping symbols and apply order of operations

## Operations with Whole

## Numbers

Find squares/related square roots; cubes/related cube roots
Use index notation for positive
integral indices
Determine and apply tests of divisibility
Express a number as a product of

## its prime factors

Divide two- or three-digit numbers
by a two-digit number

## Fractions, Decimals and

## Percentages

Perform operations with fractions, decimals and mixed numerals
Use ratios and rates to solve problems

## Consumer Arithmetic

Solve simple consumer problems including those involving earning and spending money
Calculate simple interest and find compound interest using a calculator and table of values

## Probability

Determine relative frequencies to estimate probabilities
Determine theoretical probabilities

## Rational Numbers

Express recurring decimals as fractions
Round numbers to a specified number of significant figures
Convert rates from one set of units to another

## Consumer Arithmetic

Use compound interest formula Solve consumer arithmetic problems involving compound interest, depreciation, successive discounts

## § Real Numbers

Use integers and fractions for index notation

## § Real Numbers

Define the system of real numbers distinguishing between rational and irrational numbers Perform operations with surds Convert between surd and index form

## Probability

Solve probability problems including two-stage and compound events
$\S$ - recommended topics for students who are following the 5.2 pathway but intend to study the Stage 6 Mathematics course

## Scope and Continuum of Key Ideas: Patterns and Algebra



Scope and Continuum of Key Ideas: Patterns and Algebra

Stage 4

## Algebraic Techniques

Use letters to represent numbers
Translate between words and algebraic symbols and between algebraic symbols and words
Recognise and use simple equivalent algebraic expressions

## Number Patterns

Create, record and describe number patterns using words
Use algebraic symbols to translate descriptions of number patterns
Represent number pattern relationships as points on a grid

## Algebraic Techniques

Use the algebraic symbol system to simplify, expand and factorise simple algebraic expressions Substitute into algebraic expressions
Solve linear equations and word problems using algebra

## Algebraic Techniques

Apply the index laws to simplify algebraic expressions (positive integral indices only)

## Coordinate Geometry

Use a diagram to determine midpoint lenoth and gradi an midpoint formulae interval joining two points on the number plane
Graph linear and simple non-linear relationships from equations

## Coordinate Geometry

Use distance, gradient and Apply the gradient/intercept form
to interpret and graph straight lines

## Algebraic Techniques

Simplify, expand and factorise algebraic expressions including those involving fractions or with negative and/or fractional indices
Solve linear and simple quadratic equations of the form $a x^{2}=c$
Solve quadratic equations by factorising, completing the square, or using the quadratic formula
Solve linear inequalities

Solve simultaneous equations using graphical and analytical methods for simple examples

Graph and interpret linear relationships created from simple number patterns and equations

Linear Relationships
Interpret the number plane and locate ordered pairs

Stage 5.3
Stage 5.2

## 3

## Scope and Continuum of Key Ideas: Data



## Scope and Continuum of Key Ideas: Data

## Data Representation

Draw, read and interpret graphs (line, sector, travel, step, conversion, divided bar, dot plots and stem-and-leaf plots), tables and charts
Distinguish between types of variables used in graphs Identify misrepresentation of data in graphs

## Data Representation and Analysis

Construct frequency tables for grouped data

## Data Analysis and Evaluation

Find mean and modal class for grouped data

Determine cumulative frequency quartiles of a set of scores

Construct and interpret box-andwhisker plots
Find median using a cumulative frequency table or polygon

Find the standard deviation of a set of scores using a calculator
Use the terms 'skew' and
'symmetrical' to describe the shape
of a distribution

Scope and Continuum of Key Ideas: Measurement

|  | Early Stage 1 | Stage 1 | Stage 2 | Stage 3 |
| :---: | :---: | :---: | :---: | :---: |
|  | Identify and describe the attribute of length <br> Compare lengths directly by placing objects side-by-side and aligning the ends <br> Record comparisons informally | Use informal units to estimate and measure length and distance by placing informal units end-to-end without gaps or overlaps <br> Recognise the need for metres and centimetres, and use them to estimate and measure length and distance <br> Record measurements by referring to the number and type of informal or formal units used | Estimate, measure, compare and record lengths and distances using metres, centimetres and/or millimetres <br> Convert between metres and centimetres, and centimetres and millimetres <br> Estimate and measure the perimeter of two-dimensional shapes <br> Record lengths and distances using decimal notation to two places | Select and use the appropriate unit and device to measure lengths, distances and perimeters <br> Convert between metres and kilometres; millimetres, centimetres and metres <br> Calculate and compare perimeters of squares, rectangles and equilateral and isosceles triangles <br> Record lengths and distances using decimal notation to three places |
| O L L | Identify and describe the attribute of area <br> Estimate the larger of two areas and compare using direct comparison <br> Record comparisons informally | Use appropriate informal units to estimate and measure area <br> Compare and order two or more areas <br> Record measurements by referring to the number and type of informal units used | Recognise the need for square centimetres and square metres to measure area <br> Estimate, measure, compare and record areas in square centimetres and square metres | Select and use the appropriate unit to calculate area <br> Recognise the need for square kilometres and hectares <br> Develop formulae in words for finding area of squares, rectangles and triangles |
|  | Identify and describe the attributes of volume and capacity <br> Compare the capacities of two containers using direct comparison Compare the volumes of two objects by direct observation <br> Record comparisons informally | Use appropriate informal units to estimate and measure volume and capacity <br> Compare and order the capacities of two or more containers and the volumes of two or more models or objects <br> Record measurements by referring to the number and type of informal units used | Recognise the need for a formal unit to measure volume and capacity <br> Estimate, measure, compare and record volumes and capacities using litres and millilitres <br> Measure the volume of models in cubic centimetres <br> Convert between litres and millilitres | Select the appropriate unit to measure volume and capacity <br> Recognise the need for cubic metres <br> Estimate and measure the volume of rectangular prisms <br> Determine the relationship between cubic centimetres and millilitres <br> Record volume and capacity using decimal notation to three decimal places |
| ¢ | Identify and describe the attribute of mass <br> Compare the masses of two objects by pushing, pulling or hefting or using an equal arm balance <br> Record comparisons informally | Estimate and measure the mass of an object using an equal arm balance and appropriate informal units <br> Compare and order two or more objects according to mass <br> Record measurements by referring to the number and type of informal units used | Recognise the need for a formal unit to measure mass <br> Estimate, measure, compare and record masses using kilograms and grams | Select and use the appropriate unit and device to measure mass <br> Recognise the need for tonnes <br> Convert between kilograms and grams and between kilograms and tonnes <br> Record mass using decimal notation to three decimal places |
| $\begin{aligned} & \text { © } \\ & \underset{\sim}{G} \end{aligned}$ | Describe the duration of events using everyday language <br> Sequence events in time <br> Name days of the week and seasons | Use informal units to measure and compare the duration of events Name and order the months and seasons of the year Identify the day and date on a calendar <br> Tell time on the hour and half-hour on digital and analog clocks | Recognise the coordinated movements of the hands on a clock <br> Read and record time using digital and analog notation <br> Convert between units of time | Convert between am/pm notation and 24 -hour time <br> Compare various time zones in Australia, including during daylight saving <br> Draw and interpret a timeline using a scale |
|  | Tell time on the hour on digital and analog clocks |  | Read and interpret simple timetables, timelines and calendars | Use timetables involving 24-hour time |

## Scope and Continuum of Key Ideas: Measurement

## Perimeter and Area

Describe the limits of accuracy of measuring instruments

Convert between metric units of length

Develop formulae and use to find the area and perimeter of triangles, rectangles and parallelograms
Find the areas of simple composite figures
Investigate and find the area and circumference of circles
Convert between metric units of area

## Surface Area and Volume

Find the surface area of rectangular and triangular prisms

Find the volume of right prisms and cylinders
Convert between metric units of volume

## Perimeter and Area

Apply Pythagoras' theorem

## Perimeter and Area

Develop formulae and use to find the area of rhombuses, trapeziums and kites
Find the area and perimeter of simple composite figures consisting of two shapes including quadrants and semicircles

## Perimeter and Area

Find area and perimeter of more complex composite figures

## Surface Area and Volume

Find surface area of cylinders and composite solids

Surface Area and Volume
Apply formulae for the surface area of pyramids, right cones and spheres

Explore and use similarity relationships for area and volume
Find the volume of pyramids, cones, spheres and composite solids

## Trigonometry

Solve further trigonometry problems including those involving three-figure bearings

## § Trigonometry

Determine the exact trigonometric ratios for $30^{\circ}, 45^{\circ}, 60^{\circ}$

Apply relationships in trigonometry for complementary angles and tan in terms of sin and cos

Determine trigonometric ratios for obtuse angles
Sketch sine and cosine curves
Explore trigonometry with non-right-angled triangles: sine rule, cosine rule and area rule

Solve problems involving more than one triangle using trigonometry

## Time

Perform operations involving time units

Use international time zones to compare times
Interpret a variety of tables and charts related to time

## Trigonometry

Use trigonometry to find sides and angles in right-angled triangles

Solve problems involving angles of elevation and angles of depression from diagrams
$\S$ - recommended topics for students who are following the 5.2 pathway but intend to study the Stage 6 Mathematics course

Scope and Continuum of Key Ideas: Space and Geometry

|  | Early Stage 1 | Stage 1 | Stage 2 | Stage 3 |
| :---: | :---: | :---: | :---: | :---: |
|  | Manipulate and sort threedimensional objects found in the environment <br> Describe features of threedimensional objects using everyday language <br> Use informal names for threedimensional objects | Name, describe, sort and model cones, cubes, cylinders, spheres and prisms <br> Recognise three-dimensional objects in pictures and the environment, and presented in different orientations <br> Recognise that three-dimensional objects look different from different views | Name, describe, sort, make and sketch prisms, pyramids, cylinders, cones and spheres <br> Create nets from everyday packages <br> Describe cross-sections of threedimensional objects | Identify three-dimensional objects, including particular prisms and pyramids, on the basis of their properties <br> Construct three-dimensional models given drawings of different views |
| $\begin{array}{r}0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \hline 1 \\ 0 \\ 0 \\ 0 \\ \hline\end{array}$ | Manipulate, sort and describe twodimensional shapes <br> Identify and name circles, squares, triangles and rectangles in pictures and the environment, and presented in different orientations Represent two-dimensional shapes using a variety of materials <br> Identify and draw straight and curved lines | Identify, name, compare and represent hexagons, rhombuses and trapeziums presented in different orientations <br> Make tessellating designs using flips, slides and turns Identify a line of symmetry <br> Identify and name parallel, vertical and horizontal lines <br> Identify corners as angles <br> Compare angles by placing one angle on top of another | Identify and name pentagons, octagons and parallelograms presented in different orientations Compare and describe special groups of quadrilaterals <br> Make tessellating designs by reflecting, translating and rotating <br> Find all lines of symmetry for a two-dimensional shape <br> Recognise openings, slopes and turns as angles <br> Describe angles using everyday language and the term 'right' <br> Compare angles using informal means | Identify right-angled, isosceles, equilateral and scalene triangles <br> Identify and draw regular and irregular two-dimensional shapes Identify and name parts of a circle <br> Enlarge and reduce shapes, pictures and maps <br> Identify shapes that have rotational symmetry <br> Classify angles as right, acute, obtuse, reflex, straight or a revolution <br> Measure in degrees and construct angles using a protractor |
| $\begin{aligned} & \text { 들 } \\ & \text { "등 } \\ & 0 \\ & 0 \end{aligned}$ | Give and follow simple directions <br> Use everyday language to describe position | Represent the position of objects using models and drawings <br> Describe the position of objects using everyday language, including 'left' and 'right' | Use simple maps and grids to represent position and follow routes <br> Determine the directions N, S, E and W; NE, NW, SE and SW, given one of the directions <br> Describe the location of an object on a simple map using coordinates or directions | Interpret scales on maps and plans Make simple calculations using scale |

## Scope and Continuum of Key Ideas: Space and Geometry

## Properties of Solids

Determine properties of three-
dimensional objects
Investigate Platonic solids
Investigate Euler's relationship for
convex polyhedra
Make isometric drawings

## Properties of Geometrical <br> Figures

Classify, construct and determine
properties of triangles and
quadrilaterals

Investigate similar figures and
interpret and construct scale
drawings
Identify congruent figures

Complete simple numerical exercises based on geometrical properties

## Angles

Classify angles and determine
angle relationships
Construct parallel and
perpendicular lines and determine
associated angle properties

## Properties of Geometrical Figures

Verify the properties of special quadrilaterals using congruent

Establish sum of exterior angles result and sum of interior angles result for polygons
triangles

Identify similar triangles and describe their properties Apply tests for congruent triangles

Use simple deductive reasoning in numerical and non-numerical problems

## § Deductive Geometry

Use deductive geometry to prove properties of special triangles and quadrilaterals

Construct geometrical arguments using similarity tests for triangles

Construct proofs of geometrical relationships involving congruent or similar triangles

## \# Circle Geometry

Deduce chord, angle, tangent and secant properties of circles

## \# - optional topics

§ - recommended topics for students who are following the 5.2
pathway but intend to study the Stage 6 Mathematics course

## Content Presentation

The sections that follow contain the content for Early Stage 1 to Stage 4 so that teachers can meet the learning needs of students in the primary school years. Within each strand and substrand or topic, the outcomes, key ideas, content, background information, and advice about language are presented in tables as follows. The content is comprised of the statements of knowledge and skills in the left hand column and the statements about Working Mathematically in the right hand column.

For Stages 2 and 3, there are some substrands that contain the development of several concepts. To enable ease of programming, the content has been separated into two units. The first unit typically contains early concept development and the second unit continues with further development of the concepts.

## Stage

## Substrand

## Outcome Code

A Statement of the outcome.

## Knowledge and Skills

## Students learn about

A set of statements related to the knowledge and skills students need to understand and apply in order to achieve the outcome.
These are generally presented as a hierarchy of concept development; however, separate statements would typically be grouped and addressed together when planning teaching and learning experiences.
The content is written for a whole Stage that would typically span two years of schooling.

## Key Ideas

A list of the key ideas to be addressed that summarise the content statements listed below in both the left and right columns. These are also listed on the Scope and Continuum.

## Working Mathematically

## Students learn to

A sample set of statements that incorporate Working Mathematically processes into the knowledge and skills listed in the left hand column.
Teachers are encouraged to extend this list of statements by creating their own Working Mathematically experiences for students to engage with each of the five processes (Questioning, Applying Strategies, Communicating, Reasoning and Reflecting).

Understanding is encompassed in the development of concepts and processes in both of these columns.

## Background Information

Information that provides background knowledge for teachers to assist with planning programs of study for students.

Some links with other substrands and strands have been included. Others are incorporated in the Teaching and Learning Units.

## Language

Advice about language and literacy that may assist student engagement and understanding of the content in the unit.

A list of recommended terminology is included in the Teaching and Learning Units that are not part of the syllabus.

Stage 4 content has been included in this Mathematics $K-6$ Sylabus to support the learning needs of students who have achieved Stage 3 outcomes during the primary years. Enrichment topics, such as those listed in the Additional Content on p 10, and the following Stage 4 topics are recommended for these students: NS4.1 Operations with Whole Numbers on p 57, DS4.1 Data Representation on p 89, MS4.3 Time on p 116.

## Working Mathematically

Working Mathematically encompasses five interrelated processes. These processes come into play when developing new skills and concepts and also when applying existing knowledge to solve routine and non-routine problems both within and beyond mathematics. At times the focus may be on a particular process or group of processes, but often the five processes overlap. While this strand has a set of separate outcomes, it is integrated into the content of each of the five content strands in the syllabus.

Working Mathematically provides opportunities for students to engage in genuine mathematical activity and to develop the skills to become flexible and creative users of mathematics.

The five processes for Working Mathematically are:
Questioning Students ask questions in relation to mathematical situations and their mathematical experiences. Encouraging students to ask questions builds on and stimulates their curiosity and interest in mathematics. 'I wonder if' and 'what if' types of questions encourage students to make conjectures and/or predictions.

Applying Strategies Students develop, select and use a range of strategies, including the selection and use of appropriate technology, to explore and solve problems.

Communicating Students develop and use appropriate language and representations to formulate and express mathematical ideas in written, oral and diagrammatic form.

Reasoning Students develop and use processes for exploring relationships, checking solutions and giving reasons to support their conclusions. Students also need to develop and use logical reasoning, proof and justification.

Reflecting Students reflect on their experiences and critical understanding to make connections with, and generalisations about, existing knowledge and understanding. Students make connections with the use of mathematics in the real world by identifying where, and how, particular ideas and concepts are used.

Examples of learning experiences for each of the processes for Working Mathematically are embedded in the righthand column of the content for each outcome in the Number, Patterns and Algebra, Data, Measurement, and Space and Geometry strands.

## Number

The skills developed in the Number strand are fundamental to all other strands of this Mathematics syllabus and are developed across the Stages from Early Stage 1 to Stage 5.3. Numbers, in their various forms, are used to quantify and describe the world. From Early Stage 1 there is an emphasis on the development of number sense and confidence and competence in using mental, written and calculator techniques for solving appropriate problems. Algorithms are introduced after students have gained a firm understanding of basic concepts including place value, and have developed mental strategies for computing with two- and three-digit numbers. Approximation is important and the systematic use of estimation is to be encouraged at all times. Students should always check that their answers 'make sense' in the context of the problems they are solving.

The use of mental computation strategies should be developed at all Stages. Calculators can be used to investigate number patterns and relationships and facilitate the solution of real problems with measurements and quantities not easy to handle with mental or written techniques.

The Number strand for Early Stage 1 to Stage 3 is organised into five substrands:

- Whole Numbers
- Addition and Subtraction
- Multiplication and Division
- Fractions and Decimals
- Chance.

Whole Numbers includes counting strategies, number relationships and the concept of place value. The operations are paired in the substrands Addition and Subtraction, and Multiplication and Division, to emphasise the importance of developing awareness of the inverse relationships between these operations.

In Fractions and Decimals, students are introduced to the concept of a fraction through everyday experiences. Development of the idea of division of a whole and collections of objects into equal parts leads to equivalence relationships and simple operations including addition and subtraction of fractions with denominators that are multiples of each other and multiplication of fractions by whole numbers. Students also develop an understanding of decimals and perform calculations with decimals up to three-decimal places. Percentages are introduced to enable interpretation of their use in everyday contexts.

The substrand Chance has been included from Stage 1 to enable the development of understanding of chance concepts from an early age. Early emphasis in the Chance substrand is on understanding the idea of chance and the use of informal language associated with chance. The understanding of chance situations is further developed through the use of simple experiments which produce data so that students can make comparisons of the likelihood of events occurring and begin to order chance expressions on a scale from zero to one.

Development of an understanding of the monetary system and computation with money is integrated into the substrands of Whole Numbers, Addition and Subtraction, Multiplication and Division, and Fractions and Decimals.

This section presents the outcomes, key ideas, knowledge and skills, and Working Mathematically statements from Early Stage 1 to Stage 3 in each substrand. The Stage 4 content is presented in the topics: Whole Numbers; Integers; Fractions, Decimals and Percentages; and Probability.

## Early Stage 1

## Whole Numbers

## NES1.1

Counts to 30 , and orders, reads and represents numbers in the range 0 to 20

## Knowledge and Skills

## Students learn about

- counting forwards to 30 , from a given number
- counting backwards from a given number, in the range 0 to 20
- identifying the number before and after a given number
- counting with one-to-one correspondence
- reading and writing numbers to at least 20 , including zero
- recognising a dot pattern instantly for numbers up to seven (subitising)
- representing numbers to at least 20 using numerals, words, symbols and objects (including fingers)
- comparing and ordering numbers or groups of objects
- making and recognising different visual arrangements for the same number

- using 5 as a reference in forming numbers from 6 to 10 eg 'Six is one more than five.'
- using 10 as a reference in forming numbers from 11 to 20 eg 'Thirteen is three more than ten.'
- reading and using the ordinal names to at least 'tenth'
- recognising that there are different coins and notes in our monetary system
- using the language of money in everyday contexts eg coin, note, cents, dollars


## Key Ideas

Count forwards to 30 , from a given number
Count backwards from a given number, in the range 0 to 20
Compare, order, read and represent numbers to at least 20
Read and use the ordinal names to at least 'tenth'
Use the language of money

## Working Mathematically

## Students learn to

- ask questions involving counting numbers to at least 20 eg 'How many pencils are in the tin?' (Questioning)
- apply counting strategies to solve simple everyday problems (Applying Strategies)
- communicate an understanding of number using everyday language, actions, materials and informal recordings (Communicating)
- justify answers by demonstrating the process used (Applying Strategies, Reasoning)
- recognise numbers in a variety of contexts, including on classroom charts, a calculator, shop cash register, computer keyboard and telephone (Reflecting)
- count rhythmically to identify number patterns eg stressing every second number (Applying Strategies)
- estimate the number of objects in a group of up to 20 objects, and count to check (Reflecting, Applying Strategies)
- exchange money for goods in a play situation (Reflecting)


## Background Information

At this Stage, the expectation is that students count to 30. Many classes have between 20 and 30 students and it is a common activity to count the number of students. Students will also encounter numbers up to 31 in calendars.
These numbers are only guides and should be adapted to suit the needs of individual students.
Counting is an important component of number and the early learning of operations. There is a distinction between counting by rote and counting with understanding.
Regularly counting forwards and backwards from a given number will familiarise students with the sequence.

Counting with understanding involves counting with one-toone correspondence and developing a sense of the size of numbers, their order and relationships.
Representing numbers in a variety of ways is essential for developing number sense.
The teen numbers are often the most difficult for students. The oral language pattern of teen numbers is the reverse of the usual pattern of 'tens first and then ones'. Consequently some teachers prefer to teach the teen numbers after first teaching the numbers 0 to 10 and 20 to 30 .
Subitising involves immediately recognising the number of objects in a small collection without having to count the objects.

## Language

Students may use incorrect terms since they are frequently used in everyday language eg 'How much did you get?' rather than
'How many did you get?' when referring to a score in a game.

## Stage 1

## Whole Numbers

## NS1.1 - Unit 1 (two-digit numbers)

Counts, orders, reads and represents two- and three-digit numbers

## Key Ideas

Count forwards and backwards by ones, twos and fives Count forwards and backwards by tens, on and off the decade Read, order and represent two-digit numbers
Read and use the ordinal names to at least 'thirty-first'
Sort, order and count money using face value

## Working Mathematically

## Students learn to

- ask questions involving two-digit numbers eg 'Why are the houses on either side of my house 32 and 36?' (Questioning)
- interpret numerical information from texts and in other contexts (Communicating)
- give reasons for placing a set of numbers in a particular order (Communicating, Reasoning)
- recognise and explain number patterns eg odds and evens, numbers ending with five (Communicating, Reflecting)
- use number patterns to assist with counting (Applying Strategies, Reflecting)
- use mental grouping to count and to assist with estimating the number of items in large groups (Applying Strategies)
- solve simple everyday problems using problem-solving strategies, including:
- trial and error
- drawing a diagram
(Applying Strategies, Communicating)
- determine whether there is enough money to buy a particular item (Applying Strategies)


## Background Information

The needs of students are to be considered when determining the appropriate range of two- and three-digit numbers.

By developing a variety of counting strategies and ways to combine quantities, students recognise that using strategies other than counting by ones is more efficient to count collections.

## Language

Students should be made aware that bus and telephone numbers are said differently from ordinary numbers.
Ordinal names may be confused with fraction names eg 'the third' relates to order but 'a third' is a fraction.

The word 'round' has different meanings in different contexts and some students may confuse it with the word 'around'.

## Stage 1

## Whole Numbers

## NS1.1 - Unit 2 (three-digit numbers)

Counts, orders, reads and represents two- and three-digit numbers

## Key Ideas

Read, order and represent three-digit numbers
Sort, order and count money using face value

## Working Mathematically

## Students learn to

- ask questions involving three-digit numbers eg 'Why is 153 less than 163?' (Questioning)
- identify some of the ways numbers are used in our lives (Reflecting)
- interpret numerical information from factual texts and in other contexts (Communicating)
- give reasons for placing a set of numbers in a particular order (Communicating, Reasoning)
- recognise and explain number patterns eg odds and evens, numbers ending with five and zero (Communicating, Reflecting)
- use number patterns to assist with counting (Applying Strategies, Reflecting)
- use mental grouping to count and to assist with estimating the number of items in large groups (Applying Strategies)
- make the largest and smallest number given any three digits (Applying Strategies)
- solve simple everyday problems using problem-solving strategies, including:
- trial and error
- drawing a diagram
(Applying Strategies, Communicating)
- determine whether there is enough money to buy a particular item (Applying Strategies)
- recognise that there are:

100 cents in \$1,
200 cents in $\$ 2, \ldots$ (Reflecting)

## Background Information

Students need to learn correct rounding of numbers based on the convention of rounding up if the last digit is five or more and leaving the number if the last digit is zero to four.

One cent and two-cent coins were withdrawn by the Australian Government in 1990. Prices can still be expressed in one-cent increments but the final bill is rounded to the nearest five cents. In this context, rounding is different to normal conventions in that totals ending in $3,4,6$, and 7 are rounded to the nearest 5 cents, and totals ending in $8,9,1$, and 2 are rounded to the nearest 0 cents.

## Language

The word 'and' is used when reading a number or writing it in words eg five hundred and sixty-three.

Stage 2

## Whole Numbers

## NS2. 1

Counts, orders, reads and records numbers up to four digits

## Key Ideas

Use place value to read, represent and order numbers up to four digits
Count forwards and backwards by tens or hundreds, on and off the decade

## Knowledge and Skills

## Students learn about

- representing numbers up to four digits using numerals, words, objects and digital displays
- identifying the number before and after a given two-, three- or four-digit number
- applying an understanding of place value and the role of zero to read, write and order numbers up to four digits
- stating the place value of digits in two-, three- or four-digit numbers eg 'in the number 3426, the 3 represents 3000 or 3 thousands'
- ordering a set of four-digit numbers in ascending or descending order
- using the symbols for 'is less than' $(<)$ and 'is greater than' $(>)$ to show the relationship between two numbers
- counting forwards and backwards by tens or hundreds, on and off the decade
eg 1220, 1230, 1240 (on the decade); $423,323,223$ (off the decade)
- recording numbers up to four digits using expanded notation eg $5429=5000+400+20+9$
- rounding numbers to the nearest ten, hundred or thousand when estimating


## Background Information

Students should be encouraged to develop different counting strategies eg if they are counting a large number of shells they can count out groups of ten and then count the groups.
The place value of digits in various numerals is investigated. Students should understand, for example, that the five in 35 represents five ones but the 5 in 53 represents five tens.

The convention for writing numbers of more than four digits requires that they have a space (and not a comma) to the left of each group of three digits, when counting from the Units column.

## Language

The word 'and' is used between the hundreds and the tens when reading a number, but not between other places eg three thousand, six hundred and sixty-three.

The word 'round' has different meanings in different contexts eg the plate is round, round 23 to the nearest ten.
The word 'place' has different meanings in everyday language to those used in a mathematical context.

## Stage 3

## Whole Numbers

## NS3. 1

Orders, reads and writes numbers of any size

## Key Ideas

Read, write and order numbers of any size using place value Record numbers in expanded notation
Recognise the location of negative numbers in relation to zero Identify differences between Roman and Hindu-Arabic counting systems

## Working Mathematically

## Students learn to

- ask questions that extend understanding of numbers eg 'What if ...?' (Questioning)
- use large numbers in real-life situations eg population, money applications (Reflecting, Applying Strategies)
- interpret information from the Internet, media, environment and other sources that use large numbers (Communicating)
- investigate negative numbers and the number patterns created when counting backwards on a calculator (Applying Strategies)
- link negative numbers with subtraction (Reflecting)
- interpret negative whole numbers in everyday contexts eg temperature (Communicating, Reflecting)
- record numerical data in a simple spreadsheet (Applying Strategies)
- apply strategies to estimate large quantities (Applying Strategies)


## Background Information

The convention for writing numbers of more than four digits requires that they have a space (and not a comma) to the left of each group of three digits, when counting from the Units column.
Students need to develop an understanding of place value relationships such as 10 thousand $=100$ hundreds = 1000 tens $=10000$ ones.

The abbreviation K comes from the Greek word khilioi meaning thousand. It is used in many job advertisements (eg a salary of $\$ 70 \mathrm{~K}$ ) and as an abbreviation for the size of computer files eg 26K (kilobytes).
When identifying Roman Numerals in everyday contexts it needs to be noted that the number four is sometimes represented using IIII instead of IV.

## Early Stage 1

## Addition and Subtraction

## NES1. 2

Combines, separates and compares collections of objects, describes using everyday language and records using informal methods

## Key Ideas

Combine groups to model addition
Take part of a group away to model subtraction
Compare groups to determine 'how many more'
Record addition and subtraction informally

## Knowledge and Skills

## Students learn about

- combining two or more groups of objects to model addition
- separating and taking part of a group of objects away to model subtraction
- comparing two groups of objects to determine 'how many more'
- creating combinations for numbers to at least 10 eg 'How many more make ten?'

- describing the action of combining, separating or comparing using everyday language eg makes, join, and, get, take away, how many more, altogether
- counting forwards by ones to add and backwards by ones to subtract
- recording addition and subtraction informally using drawings, numerals and words


## Working Mathematically

## Students learn to

- pose 'how many' questions that can be solved using addition and subtraction (Questioning)
- use concrete materials, including fingers, to model and solve simple addition and subtraction problems (Applying Strategies)
- solve simple everyday problems using problem-solving strategies that include 'acting it out' (Applying Strategies)
- use visualisation of numbers to assist with addition and subtraction (Applying Strategies)
- apply strategies that have been demonstrated by other students (Applying Strategies, Reflecting)
- use simple computer graphics to represent numbers and their combinations to at least 10 (Applying Strategies)
- explain or demonstrate how an answer was obtained (Applying Strategies, Communicating, Reasoning)
- describe what happened to a group when it was added to or subtracted from (Communicating, Reflecting)


## Background Information

Addition and Subtraction should move from counting and combining perceptual objects, to using numbers as replacements for completed counts with mental strategies, to recordings that support mental strategies (such as jump or split, partitioning or compensation).
At this Stage, addition and subtraction problems should be related to real-life experiences that involve the manipulation of objects.
Subtraction typically covers three different situations

- 'taking away' from a group
- 'comparing' two groups
- finding 'how many more'.

Students should be confident with the taking away from a group before being introduced to 'comparing' two groups.
Students should be able to compare groups of objects by using one-to-one correspondence before being asked to find out how many more or how many less there are in a group.
Modelling, drawing and writing mathematical problems should be encouraged at this Stage. Formal writing of number sentences is introduced at the next Stage.

## Language

Some students may need assistance when two tenses are used within the one problem eg 'I had six beans and took away four. How many do I have?'
The word 'difference' has a specific meaning in this context, referring to the numeric value of the group. In everyday language it can refer to any attribute.

The word 'left' can be ambiguous eg 'There were five children in the room. Three went to lunch. How many left?' Is the question asking how many children are remaining in the room or how many children went to lunch?

## Stage 1

## Addition and Subtraction

## NS 1.2

Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and twodigit numbers

## Key Ideas

Model addition and subtraction using concrete materials
Develop a range of mental strategies and informal recording methods for addition and subtraction
Record number sentences using drawings, numerals, symbols and words

## Working Mathematically

## Students learn to

- recall addition and subtraction facts for numbers to at least 20 (Applying Strategies)
- use simple computer graphics to represent numbers and their combinations to at least 20 (Applying Strategies)
- pose problems that can be solved using addition and subtraction, including those involving money ( Questioning)
- select and use a variety of strategies to solve addition and subtraction problems (Applying Strategies)
- check solutions using a different strategy (Applying Strategies, Reasoning)
- recognise which strategy worked and which did not work (Reasoning, Reflecting)
- explain why addition and subtraction are inverse (opposite) operations (Communicating, Reasoning)
- explain or demonstrate how an answer was obtained for addition and subtraction problems
eg showing how the answer to $15+8$ was obtained using a jump strategy on an empty number line

(Communicating, Reasoning)
- use a variety of own recording strategies (Applying Strategies, Communicating)
- recognise equivalent amounts of money using different denominations eg 50c can be made up of two 20c coins and a 10c coin (Reflecting, Applying Strategies)
- calculate mentally to give change (Applying Strategies)
- using related addition and subtraction number facts to at least 20 eg $15+3=18$, so $18-15=3$
- using concrete materials to model addition and subtraction problems involving one- and two-digit numbers
- using bundling of objects to model addition and subtraction with trading
- using a range of strategies for addition and subtraction of two-digit numbers, including
- split strategy
- jump strategy (as recorded on an empty number line)
- performing simple calculations with money including finding change and rounding to the nearest 5 c


## Stage 1

## Addition and Subtraction (continued)

## Background Information

It is appropriate for students at this Stage to use concrete materials to model and solve problems, for exploration and for concept building. Concrete materials may also help in explanations of how solutions were arrived at.
Addition and Subtraction should move from counting and combining perceptual objects, to using numbers as replacements for completed counts with mental strategies, to recordings that support mental strategies (such as jump or split, partitioning or compensation).
At this Stage, students develop a range of strategies to aid quick recall of number facts and to solve addition and subtraction problems. Students should be encouraged to explain their strategies and invent ways of recording their actions. It is also important to discuss the merits of various strategies in terms of practicality and efficiency.
In performing a subtraction, students could use 'counting on or back' from one number to find the difference.
The 'counting on or back' type of subtraction is more difficult for students to grasp. Nevertheless, it is important to encourage students to use the 'counting on' strategy as a method of solving comparison problems after they are confident with the 'take away' type.

## Jump strategy on a number line:

An addition or subtraction strategy in which the student places the first number on an empty number line and then counts forward or backwards firstly by tens and then by ones to perform a calculation. (The number of jumps will reduce with increased understanding.)
eg $46+33$
Method 1:

Method 2:


## Language

Some students may need assistance when two tenses are used within the one problem, eg 'I had six beans and took away four. How many do I have?'
The word 'difference' has a specific meaning in this context, referring to the numeric value of the group. In everyday language it can refer to any attribute.

eg 79-33
Method 1


Method 2


## Split strategy:

An addition or subtraction strategy in which the student separates the tens from the units and adds or subtracts each separately before combining to obtain the final answer.
eg $46+33$
$=40+6+30+3$
$=40+30+6+3$
$=70+9$
$=79$
eg 79-33
$=70+9-30-3$
$=70-30+9-3$
$=40+6$
$=46$

Students need to understand that the need to carry out subtraction can be indicated by a variety of language structures. The language used in the 'comparison' type of subtraction is quite different to that used in the 'take away' type.

Stage 2

## Addition and Subtraction

## NS2. 2

Uses mental and written strategies for addition and subtraction involving two-, three- and four-digit numbers

## Key Ideas

Use a range of mental strategies for addition and subtraction involving two-, three- and four-digit numbers
Explain and record methods for adding and subtracting
Use a formal written algorithm for addition and subtraction

## Working Mathematically

## Students learn to

- pose problems that can be solved using addition and subtraction, including those involving money ( Questioning)
- ask 'What is the best method to find a solution to this problem?' ( Questioning)
- select and use mental, written or calculator methods to solve addition and subtraction problems (Applying Strategies)
- solve a variety of problems using problem-solving strategies, including:
- trial and error
- drawing a diagram
- working backwards
- looking for patterns
- using a table
(Applying Strategies, Communicating)
- use estimation to check solutions to addition and subtraction problems, including those involving money (Reflecting, Applying Strategies)
- check the reasonableness of a solution to a problem by relating it to an original estimation (Reasoning)
- check solutions using the inverse operation or a different method (Applying Strategies, Reasoning)
- explain how an answer was obtained for an addition or subtraction problem (Communicating, Reasoning)
- reflect on own method of solution for a problem, considering whether it can be improved (Reflecting)
- use a calculator to generate number patterns, using addition and subtraction (Applying Strategies)
- adding and subtracting two or more numbers, with and without trading, using concrete materials and recording their method
- using a formal written algorithm and applying place value to solve addition and subtraction problems, involving two-, three- and four-digit numbers
eg $\left.\begin{array}{rrrr}134 & 2459 & 568 & 1353 \\ & +253 & +138 & -322\end{array}\right)-168$


## Stage 2

## Addition and Subtraction (continued)

## Background Information

Students should be encouraged to estimate answers before attempting to solve problems in concrete or symbolic form.
There is still a need to emphasise mental computation even though students can now use a formal written method. The following formal methods may be used.

## Decomposition

The following example shows a suitable layout for the decomposition method.

$$
2 \frac{3}{3} 56
$$

- 1385

1071

## Equal Addends

For students who have a good understanding of subtraction, the equal addends algorithm may be introduced as an alternative, particularly where very large numbers are involved. There are several possible layouts of the method, of which the following is only one and not necessarily the best. The expression 'borrow and pay back' should not be used. 'Add ten ones' and 'add ten' is preferable.

$$
\begin{array}{r}
1 \quad 111 \\
38612 \\
-29893 \\
\hline 8719
\end{array}
$$

When developing a formal written algorithm, it will be necessary to sequence the examples to cover the range of possibilities that include with and without trading in one or more places, and one or more zeros in the first number.

## Language

Word problems requiring subtraction usually fall into two types - either 'take away' or 'comparison'. The comparison type of subtraction involves finding how many more need to be added to a group to make it equivalent to a second group, or finding the difference between two groups. Students need to be able to translate from these different language contexts into a subtraction calculation.

The word 'difference' has a specific meaning in a subtraction context.
Difficulties could arise for some students with use of the passive voice in relation to subtraction problems eg '10 takeaway 9 ' will give a different response to ' 10 was taken away from 9'.

Stage 3

## Addition and Subtraction

## NS3. 2

Selects and applies appropriate strategies for addition and subtraction with counting numbers of any size

## Key Ideas

Select and apply appropriate mental, written or calculator strategies for addition and subtraction with counting numbers of any size

## Working Mathematically

## Students learn to

- ask 'What if' questions eg 'What happens if we subtract a larger number from a smaller number on a calculator?' (Questioning)
- pose problems that can be solved using counting numbers of any size and more than one operation (Questioning)
- explain whether an exact or approximate answer is best suited to a situation (Communicating)
- use a number of strategies to solve unfamiliar problems, including:
- trial and error
- drawing a diagram
- working backwards
- looking for patterns
- using a table
- simplifying the problem
(Applying Strategies, Communicating)
- check solutions by using the inverse operation or a different method (Applying Strategies, Reasoning)
- explain how an answer was obtained for an addition or subtraction problem and justify the selected calculation method (Communicating, Reasoning)
- give reasons why a calculator was useful when solving a problem (Reasoning, Applying Strategies)
- reflect on chosen method of solution for a problem, considering whether it can be improved (Reflecting)


## Background Information

At this Stage, mental strategies need to be continually reinforced and used to check results obtained using formal algorithms.
Students may find that their own written strategies that are based on mental strategies may be more efficient than a formal written algorithm, particularly for the case of subtraction. For example $8000-673$ is easier to do mentally than by using either the decomposition or the equal addends methods.

## Mentally:

$8000=7999+1$
$7999-673=7326$
The answer will therefore be $7326+1=7327$.
This is just one way of doing this mentally: students could share possible approaches and compare them to determine the most efficient.

## Decomposition Method:

$3 Q{ }^{2}$
QQQ
$\begin{array}{r}-\quad 673 \\ \hline 7327\end{array}$

## Equal Addends Method:

$\begin{array}{r}111 \\ 8000 \\ -\quad{ }_{16} 673 \\ \hline 7327\end{array}$

## Language

Difficulties could arise for some students with use of the passive voice in relation to subtraction problems eg ' 10 take away 9 ' will give a different response to ' 10 was taken away from 9'.

## Early Stage 1

## Multiplication and Division

## NES1. 3

Groups, shares and counts collections of objects, describes using everyday language and records using informal methods

## Key Ideas

Model equal groups or rows
Group and share collections of objects equally
Record grouping and sharing informally

## Knowledge and Skills

## Students learn about

- using the term 'group' to describe a collection of objects
- using the term 'sharing' to describe the distribution of a collection of objects
- grouping and sharing using concrete materials
- modelling equal groups or equal rows
- recognising unequal groups or unequal rows
- labelling the number of objects in a group or row
- recording grouping and sharing informally using pictures, numerals and words


## Working Mathematically

## Students learn to

- pose problems that can be solved using grouping or sharing (Ouestioning)
- respond to grouping and sharing questions by drawing, making, acting, guessing and checking, and retelling (Communicating, Applying Strategies)
- describe grouping and sharing using everyday language, actions, materials and drawings (Communicating)
- explain or demonstrate how an answer was obtained (Applying Strategies, Communicating, Reasoning)


## Background Information

All activities should involve students manipulating concrete materials.
The emphasis is on understanding the modelling of groups of the same size and describing them. Students need to acquire the concept that fair sharing means all shares are equal.
After students have shared objects equally, the process can be reversed to begin to develop the link between division and multiplication. This can be done by students first sharing a group of objects and then putting back together all of the shares to form one collection.

There are two forms of division:
SHARING - How many in each group?
eg 'If twelve marbles are shared between three students, how many does each get?'
GROUPING - How many groups are there?
eg 'If I have twelve marbles and each child is to get four, how many children will get marbles?'
Finding the total number of objects that have been shared or grouped can be done incidentally, however, this is emphasised in Stage 1.

## Stage 1

## Multiplication and Division

## NS1. 3

Uses a range of mental strategies and concrete materials for multiplication and division

## Key Ideas

Rhythmic and skip count by ones, twos, fives and tens
Model and use strategies for multiplication including arrays, equal groups and repeated addition
Model and use strategies for division including sharing, arrays and repeated subtraction
Record using drawings, numerals, symbols and words

## Knowledge and Skills

## Students learn about

- counting by ones, twos, fives and tens using rhythmic or skip counting
- describing collections of objects as 'rows of' and 'groups of'
- modelling multiplication as equal groups or as an array of equal rows eg two groups of three

- finding the total number of objects using
- rhythmic or skip counting
- repeated addition
eg '5 groups of 4 is the same as $4+4+4+4+4$ '
- modelling the commutative property of multiplication eg ' 3 groups of 2 is the same as 2 groups of 3 '
- modelling division by sharing a collection of objects into equal groups or as equal rows in an array eg six objects shared between two friends

- modelling division as repeated subtraction
- recognising odd and even numbers by grouping objects into two rows
- recognising the symbols $\times, \div$ and $=$
- recording multiplication and division problems using drawings, numerals, symbols and words


## Working Mathematically

## Students learn to

- pose simple multiplication and division problems, including those involving money
(Questioning, Reflecting)
- answer mathematical problems using objects, diagrams, imagery, actions or trial-and-error (Applying Strategies)
- use a number line or hundreds chart to solve multiplication and division problems (Applying Strategies)
- use estimation to check that the answers to multiplication and division problems are reasonable (Applying Strategies, Reasoning)
- use patterns to assist counting by twos, fives or tens (Reflecting, Applying Strategies)
- describe the pattern created by modelling odd and even numbers (Communicating)
- explain multiplication and division strategies using language, actions, materials and drawings (Communicating, Applying Strategies)
- support answers to multiplication and division problems by explaining or demonstrating how the answer was obtained (Reasoning)
- recognise which strategy worked and which did not work (Reasoning, Reflecting)


## Background Information

There are two forms of division:
SHARING - How many in each group?
eg 'If twelve marbles are shared between three students, how many does each get?'
GROUPING - How many groups are there?
eg 'If I have twelve marbles and each child is to get four, how many children will get marbles?' This form of division relates to repeated subtraction.
After students have made equal groups (eg 3 groups of 4), the process can be reversed by sharing (eg share 12 between 3 ), thus linking multiplication and division.

When sharing a collection of objects into two or four groups, students may describe the groups as being one-half or onequarter of the whole collection.
An array is one of several different arrangements that can be used to model multiplicative situations involving whole numbers. An array is made by arranging a set of objects, such as counters, into columns and rows. Each column must contain the same number of objects as the other columns, and each row must contain the same number of objects as the other rows.

## Language

The term 'lots of' can be confusing to students because of its everyday use and should be avoided eg 'lots of fish in the sea'.

It is preferable that students use 'groups of' or 'rows of'.

Stage 2

## Multiplication and Division

NS2.3 - Unit 1 (multiplication and division facts)
Uses mental and informal written strategies for multiplication and division

## Knowledge and Skills

## Students learn about

- counting by threes, fours, sixes, sevens, eights or nines using skip counting
- linking multiplication and division facts using groups or
arrays
eg -•••
3 groups of 4 is 12
$3 \times 4=12$
12 shared among 3 is 4 $12 \div 3=4$
- using mental strategies to recall multiplication facts up to $10 \times 10$, including
- the commutative property of multiplication eg $7 \times 9=9 \times 7$
- using known facts to work out unknown facts eg $5 \times 5=25$ so $5 \times 6=(5 \times 5)+5$
- the relationship between multiplication facts eg 'the multiplication facts for 6 are double the multiplication facts for 3 '
- recognising and using $\div$ and $\Gamma$ to indicate division
- using mental strategies to divide by a one-digit number, including
- the inverse relationship of multiplication and division eg $63 \div 9=7$ because $7 \times 9=63$
- recalling known division facts
- relating to known division facts eg $36 \div 4$; halve 36 and halve again
- describing and recording methods used in solving multiplication and division problems
- listing multiples for a given number
- finding square numbers using concrete materials and diagrams


## Key Ideas

Develop mental facility for number facts up to $10 \times 10$
Find multiples and squares of numbers

## Working Mathematically

## Students learn to

- recall multiplication facts up to $10 \times 10$, including zero facts (Applying Strategies)
- solve a variety of problems using problem-solving strategies, including:
- trial and error
- drawing a diagram
- working backwards
- looking for patterns
- using a table
(Applying Strategies, Communicating)
- explain why a rectangular array can be read as a division in two ways by forming vertical or horizontal groups eg $12 \div 4=3$ or $12 \div 3=4$
(Reasoning, Communicating)
- check the reasonableness of a solution to a problem by relating it to an original estimation (Reasoning)
- explain how an answer was obtained and compare own method/s of solution to a problem with those of others (Communicating, Reflecting)
- use multiplication and division facts in board, card and computer games (Applying Strategies)
- apply the inverse relationship of multiplication and division to check answers eg $63 \div 9$ is 7 because $7 \times 9=63$ (Applying Strategies, Reflecting)
- create a table or simple spreadsheet to record multiplication facts (Applying Strategies)
- explain why the numbers $1,4,9,16, \ldots$ are called square numbers
(Communicating, Reasoning, Reflecting)


## Background Information

At this Stage, the emphasis in multiplication and division is on students developing mental strategies and using their own (informal) methods for recording their strategies. Comparing their method of solution with those of others, will lead to the identification of efficient mental and written strategies.
One problem may have several acceptable methods of solution.

Linking multiplication and division is an important understanding for students at this Stage. Students should come to realise that division 'undoes' multiplication and multiplication 'undoes' division. Students should be encouraged to check the answer to a division question by multiplying their answer by the divisor. To divide, students may recall division facts or transform the division into a multiplication and use multiplication facts eg $35 \div 7$ is the same as $\square \times 7=35$.

## Language

When beginning to build and read multiplication tables aloud, it is best to use a language pattern of words that relates back to concrete materials such as arrays.
As students become more confident with recalling multiplication number facts, they may use less language.

For example, ‘seven rows (or groups) of three’ becomes ‘seven threes' with the 'rows of' or 'groups of' implied. This then leads to:

- one three is three
- two threes are six
- three threes are nine, and so on.


## Stage 2

## Multiplication and Division

## NS2. 3 - Unit 2

Uses mental and informal written strategies for multiplication and division

## Key Ideas

Use mental and informal written strategies for multiplying or dividing a two-digit number by a one-digit operator
Interpret remainders in division problems
Determine factors for a given number

## Knowledge and Skills

## Students learn about

- using mental strategies to multiply a one-digit number by a multiple of $10(\mathrm{eg} 3 \times 20)$ by
- repeated addition $(20+20+20=60)$
- using place value concepts $(3 \times 2$ tens $=6$ tens $=60)$
$-\quad$ factoring $(3 \times 2 \times 10=6 \times 10=60)$
- using mental strategies to multiply a two-digit number by a one-digit number, including
- using known facts

$$
\text { eg } 10 \times 9=90 \text { so } 13 \times 9=90+9+9+9
$$

- multiplying the tens and then the units
eg $7 \times 19$ is $(7 \times 10)+(7 \times 9)=70+63=133$
- the relationship between multiplication facts
eg $23 \times 4$ is double 23 and double again
- factorising eg $18 \times 5=9 \times 2 \times 5=9 \times 10=90$
- using mental strategies to divide by a one-digit number, in problems for which answers include a remainder eg $29 \div 6$; if $4 \times 6=24$ and $5 \times 6=30$ the answer is 4 remainder 5
- recording remainders to division problems eg $17 \div 4=4$ remainder 1
- recording answers, which include a remainder, to division problems to show the connection with multiplication eg $17=4 \times 4+1$
- interpreting the remainder in the context of the word problem
- describing multiplication as the product of two or more numbers
- describing and recording methods used in solving multiplication and division problems
- determining factors for a given number eg factors of 12 are $1,2,3,4,6,12$


## Working Mathematically

## Students learn to

- pose and solve multiplication and division problems ( Questioning, Applying Strategies)
- select and use mental, written and calculator strategies to solve multiplication or division problems
eg 'to multiply by 12 , multiply by 6 and then double' (Applying Strategies)
- solve a variety of problems using problem-solving strategies, including:
- trial and error
- drawing a diagram
- working backwards
- looking for patterns
- using a table
(Applying Strategies, Communicating)
- identify the operation/s required to solve a problem (Applying Strategies)
- check the reasonableness of a solution to a problem by relating it to an original estimation (Reasoning)
- explain how an answer was obtained and compare own method/s of solution to a problem with those of others (Communicating, Reflecting)
- use multiplication and division facts in board, card and computer games (Applying Strategies)
- apply the inverse relationship of multiplication and division to check answers eg $63 \div 9$ is 7 because $7 \times 9=63$ (Applying Strategies, Reflecting)
- explain why a remainder is obtained in answers to some division problems (Communicating, Reasoning)
- apply factorisation of a number to aid mental computation eg $16 \times 25=4 \times 4 \times 25=4 \times 100=400$
(Applying Strategies)


## Background Information

At this Stage, the emphasis in multiplication and division is on students developing mental strategies and using their own (informal) methods for recording their strategies. Comparing their method of solution with those of others, will lead to the identification of efficient mental and written strategies.

One problem may have several acceptable methods of solution. Students could extend their recall of number facts beyond the multiplication facts to $10 \times 10$ by also memorising multiples of numbers such as $11,12,15,20$ and 25.

## Language

The term 'product' has a different meaning in mathematics from its everyday usage.

Stage 3

## Multiplication and Division

## NS3. 3

Selects and applies appropriate strategies for multiplication and division

## Knowledge and Skills

## Students learn about

- applying appropriate mental, written or calculator strategies to solve multiplication and division problems
- recognising and using different notations to indicate division eg $25 \div 4,4) 25, \frac{25}{4}$
- recording remainders as fractions or decimals, where appropriate eg $25 \div 4=6 \frac{1}{4}$ or 6.25
- multiplying three- and four-digit numbers by one-digit numbers using mental or written strategies

$$
\begin{array}{rlr}
\text { eg } 432 \times 5 & \begin{array}{l}
\text { (mental) } \\
= \\
\\
\\
\\
\\
\\
\\
\end{array} 20000 \times 5+30 \times 5+2 \times 5 & \text { (written) } \\
432  \tag{432}\\
& \frac{5}{2160}
\end{array}
$$

- multiplying three-digit numbers by two-digit numbers using the extended form (long multiplication) eg 521

$$
\begin{array}{r}
\times \quad 22 \\
\hline 1042 \\
10420 \\
\hline 11462
\end{array}
$$

- dividing a number with three or more digits by a singledigit divisor using mental or written strategies (mental)
eg $341 \div 4 \quad 340 \div 4=85$

$$
1 \div 4=\frac{1}{4}
$$

$$
341 \div 4=85 \frac{1}{4}
$$

$$
4 \longdiv { 3 4 1 }
$$

- using mental strategies to multiply or divide a number by 100 or a multiple of 10
- finding solutions to questions involving mixed operations eg $5 \times 4+7=27$
- determining whether a number is prime or composite by finding the number of factors eg ' 13 has two factors (1 and 13) and therefore is prime; 21 has more than two factors $(1,3,7,21)$ and therefore is composite'


## Background Information

Students could extend their recall of number facts beyond the multiplication facts to $10 \times 10$ by also memorising multiples of numbers such as $11,12,15,20$ and 25 , and/or utilise mental strategies such as ' $14 \times 6$ is 10 sixes plus 4 sixes'.
One is not a prime number because it has only one factor, itself.

## Key Ideas

Select and apply appropriate mental, written or calculator strategies for multiplication and division
Use formal written algorithms for multiplication (limit operators to two-digit numbers) and division (limit operators to single digits)
Explore prime and composite numbers

## Working Mathematically

## Students learn to

- estimate answers to problems and check to justify solutions (Applying Strategies, Reasoning)
- select an appropriate strategy for the solution of multiplication and division problems (Applying Strategies, Reflecting)
- use a number of strategies to solve unfamiliar problems, including:
- trial and error - drawing a diagram
- working backwards - looking for patterns
- simplifying the problem - using a table (Applying Strategies, Communicating)
- use the appropriate operation in solving problems in reallife situations (Applying Strategies, Reflecting)
- give a valid reason for a solution to a multiplication or division problem and check that the answer makes sense in the original situation (Communicating, Reasoning)
- use mathematical terminology and some conventions to explain, interpret and represent multiplication and division in a variety of ways (Applying Strategies, Communicating)
- use and interpret remainders in answers to division problems eg realising that the answer needs to be rounded up if the problem involves finding the number of cars needed to take 48 people to an event (Applying Strategies, Communicating)
- question the meaning of packaging statements when determining the best buy eg 4 toilet rolls for $\$ 2.95$ or 6 toilet rolls for $\$ 3.95$ (Questioning)
- determine that when a number is divided by a larger number a fraction which is less than 1 is the result (Reflecting)
- calculate averages in everyday contexts eg temperature, sport scores (Applying Strategies)
- explain why a prime number when modelled as an array has only one row (Communicating, Reflecting)

The simplest form of multiplication word problems relate to rates eg If four students earn \$3 each, how much do they have altogether? Another type of problem is related to ratio and uses language such as 'twice as many as' and 'six times as many as'. The terms rate and ratio are not introduced at this Stage, but students need to be able to interpret these problems as requiring multiplication.

## Stage 4

## Operations with Whole Numbers

## NS4. 1

Recognises the properties of special groups of whole numbers and applies a range of strategies to aid computation

## Key Ideas

Explore other counting systems
Investigate groups of positive whole numbers
Determine and apply tests of divisibility
Express a number as a product of its prime factors
Find squares/related square roots; cubes/related cube roots
Use index notation for positive integral indices
Apply mental strategies to aid computation
Divide two- or three-digit numbers by a two-digit number

## Knowledge and Skills

## Students learn about

- expressing a number as a product of its prime factors
- using index notation to express powers of numbers (positive indices only) eg $8=2^{3}$
- using the notation for square root $(\sqrt{ })$ and cube root $(\sqrt[3]{ })$
- recognising the link between squares and square roots and cubes and cube roots eg $2^{3}=8$ and $\sqrt[3]{8}=2$
- exploring through numerical examples that:
- $(a b)^{2}=a^{2} b^{2}$, eg $(2 \times 3)^{2}=2^{2} \times 3^{2}$
- $\sqrt{a b}=\sqrt{a} \times \sqrt{b}$, eg $\sqrt{9 \times 4}=\sqrt{9} \times \sqrt{4}$
- finding square roots and cube roots of numbers expressed as a product of their prime factors
- finding square roots and cube roots of numbers using a calculator, after first estimating
- identifying special groups of numbers including figurate numbers, palindromic numbers, Fibonacci numbers, numbers in Pascal's triangle
- comparing the Hindu-Arabic number system with number systems from different societies past and present
- determining and applying tests of divisibility
- using an appropriate non-calculator method to divide twoand three-digit numbers by a two-digit number
- applying a range of mental strategies to aid computation, for example
- a practical understanding of associativity and commutativity
eg $2 \times 7 \times 5=7 \times(2 \times 5)=70$
- to multiply a number by 12 , first multiply by 6 and then double the result
- to multiply a number by 13 , first multiply the number by ten and then add 3 times the number
- to divide by 20, first halve the number and then divide by 10
- a practical understanding of the distributive law eg to multiply any number by 9 first multiply by 10 and then subtract the number


## Working Mathematically

## Students learn to

- question whether it is more appropriate to use mental strategies or a calculator to find the square root of a given number ( Questioning)
- discuss the strengths and weaknesses of different number systems (Communicating, Reasoning)
- describe and recognise the advantages of the HinduArabic number system (Communicating, Reasoning)
- apply tests of divisibility mentally as an aid to calculation (Applying Strategies)
- verify the various tests of divisibility (Reasoning)


## Stage 4

## Operations with Whole Numbers (continued)

## Background Information

This work with squares and square roots links to Pythagoras' theorem in Measurement.
Calculations with cubes and cube roots may be applied in volume problems in Measurement.
The topic of special groups of numbers links with number patterning in Patterns and Algebra.
To divide two- and three-digit numbers by a two-digit number, students may be taught the long division algorithm or, alternatively, to transform the division into a multiplication.
eg (i) $88 \div 44=2$ because $2 \times 44=88$;
(ii) $356 \div 52=\square$ becomes $52 \times \square=356$. Knowing that there are two fifties in each 100, students may try 7 so that $52 \times 7=364$ which is too large.
Try $6,52 \times 6=312$.
Answer is $6 \frac{44}{52}=6 \frac{11}{13}$
Students also need to be able to express a division in the following form in order to relate multiplication and division:

$$
356=6 \times 52+44
$$

Divide by 52:

$$
\frac{356}{52}=6+\frac{44}{52}=6 \frac{44}{52}=6 \frac{11}{13}
$$

The square root sign signifies a positive number (or zero). Thus $\sqrt{9}=3$ (only). However, the two numbers whose square is 9 are $\sqrt{9}$ and $-\sqrt{9}$ ie 3 and -3 .

Figurate numbers include triangular numbers, square numbers, and pentagonal numbers.
The meaning (and possibly the derivation) of the 'radical sign' may provide an interesting historical perspective.
Number systems from different societies past and present could include Egyptian, Babylonian, Roman, Mayan, Aboriginal, and Papua-New Guinean. The differences to be compared may include those related to the symbols used for numbers and operations, the use of zero, the base system, place value, and notation for fractions.
The Internet is a source of information on number systems in use in other cultures and/or at other times in history.
The Chinese mathematician, Chu Shi-kie, wrote about the triangle result (which we now call Pascal's triangle) in 1303 at least 400 years before Pascal.

## Language

Note the distinction between the use of fewer/fewest for number of items and less/least for quantities eg 'There are fewer students in this class; there is less milk today.'

Words such as 'square' have more than one mathematical context eg draw a square; square three; find the square root of 9 . Students may need to have these differences explained.
Words such as 'product', ‘odd', 'prime' and 'power’ have different meanings in mathematics from their everyday usage. This may be confusing for some students.

Stage 4

## Integers

## NS4. 2

Compares, orders and calculates with integers

## Key Ideas

Perform operations with directed numbers
Simplify expressions involving grouping symbols and apply order of operations

## Knowledge and Skills

## Students learn about

- recognising the direction and magnitude of an integer
- placing directed numbers on a number line
- ordering directed numbers
- interpreting different meanings (direction or operation) for the + and - signs depending on the context
- adding and subtracting directed numbers
- multiplying and dividing directed numbers
- using grouping symbols as an operator
- applying order of operations to simplify expressions
- keying integers into a calculator using the +/- key
- using a calculator to perform operations with integers


## Background Information

Complex recording formats for directed numbers such as raised signs can be confusing. The following formats are recommended.

$$
\begin{aligned}
-2-3 & =-5 \\
-3+6 & =3 \\
-3+(-4) & =-3-4=-7 \\
-2-(-3) & =-2+3=1 \\
-3.25+6.83 & =3.58
\end{aligned}
$$

Brahmagupta, an Indian mathematician and astronomer (c 598 - c 665 AD ), is noted for the introduction of zero and negative numbers in arithmetic.

## Early Stage 1

## Fractions and Decimals

## NES1.4

Describes halves, encountered in everyday contexts, as two equal parts of an object

## Knowledge and Skills

## Students learn about

- sharing an object by dividing it into two equal parts eg cutting a piece of fruit into halves
- recognising that halves are two equal parts
- recognising when two parts are not halves of the one whole
- using the term 'half' in everyday situations
- recording fractions of objects using drawings eg drawing a pizza cut in half


## Key Ideas

Divide an object into two equal parts
Recognise and describe halves

## Working Mathematically

## Students learn to

- use fraction language in everyday situations eg 'one-half of a cake has been eaten' (Communicating)
- describe how to make equal parts eg describe how to cut a sandwich into halves (Communicating)
- explain the reason for dividing an object in a particular way (Communicating, Reasoning)


## Background Information

The focus on halves at this Stage is only a guide. Some students will be able to describe other fractions from everyday contexts. At this Stage, the emphasis is on dividing one whole object into two equal parts. Fairness in making equal parts is the focus. Halves can be different shapes
eg


Halves of different objects can be different sizes eg half of a sheet of art paper is larger than half of a serviette. Fractions refer to the relationship of the equal parts to the whole unit.

## Language

In everyday use, the term 'half' is sometimes used to mean one of two parts and not necessarily two equal parts eg 'I'll have the biggest half.' It is important to model and reinforce the language 'two equal parts' when describing half.

## Stage 1

## Fractions and Decimals

## NS1.4

Describes and models halves and quarters, of objects and collections, occurring in everyday situations

## Key Ideas

Model and describe a half or a quarter of a whole object
Model and describe a half or a quarter of a collection of objects Use fraction notation $\frac{1}{2}$ and $\frac{1}{4}$

## Knowledge and Skills

## Students learn about

- modelling and describing a half or a quarter of a whole object
- modelling and describing a half or a quarter of a collection of objects
- describing equal parts of a whole object or collection of objects
- describing parts of an object or collection of objects as 'about a half', 'more than a half ' or 'less than a half'
- using fraction notation for half ( $\left(\frac{1}{2}\right)$ and quarter $\left(\frac{1}{4}\right)$
- recording equal parts of a whole, and the relationship of the groups to the whole using pictures and fraction notation
eg

- identifying quarters of the same unit as being the same



## Working Mathematically

## Students learn to

- question if parts of a whole object, or collection of objects, are equal (Questioning)
- explain why the parts are equal (Communicating, Reasoning)
- use fraction language in a variety of everyday contexts eg the half-hour, one-quarter of the class (Communicating)
- recognise the use of fractions in everyday contexts eg half-hour television programs
(Communicating, Reflecting)
- visualise fractions that are equal parts of a whole eg imagine where you would cut the cake before cutting it (Applying Strategies)


## Background Information

At this Stage, fractions are used in two different ways:

- to describe equal parts of a whole, and
- to describe equal parts of a collection of objects.

Fractions refer to the relationship of the equal parts to the whole unit. When using collections to model fractions it is important that students appreciate the collection as being a 'whole' and the resulting groups as 'parts of that whole'. It should be noted that the size of the resulting fraction will depend on the size of the original whole or collection of objects.

It is not necessary for students to distinguish between the roles of the numerator and denominator at this Stage. They may use the symbol ' $\frac{1}{2}$ ' as an entity to mean 'one-half' or 'a half' and similarly for ' $\frac{1}{4}$ '.

## Language

Some students may hear 'whole' in the phase 'part of a whole' and confuse it with the term 'hole'.

At this Stage, the term 'three-quarters' may be used informally to name the remaining parts after one-quarter has been identified.

Stage 2

## Fractions and Decimals- Unit 1

## NS2.4

Models, compares and represents commonly used fractions and decimals, adds and subtracts decimals to two decimal places, and interprets everyday percentages

## Key Ideas

Model, compare and represent fractions with denominators 2, 4 and 8 , followed by fractions with denominators of 5, 10 and 100 Model, compare and represent decimals to 2 decimal places Add and subtract decimals with the same number of decimal places (to 2 decimal places)
Perform calculations with money

## Knowledge and Skills

## Students learn about

- modelling, comparing and representing fractions with denominators 2,4 and 8 by
- modelling halves, quarters and eighths of a whole object or collection of objects
- naming fractions with denominators of two, four and eight up to one whole eg $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}$
- comparing and ordering fractions with the same denominator eg $\frac{1}{8}$ is less than $\frac{3}{8}$ is less than $\frac{6}{8}$
- interpreting the denominator as the number of equal parts a whole has been divided into
- interpreting the numerator as the number of equal fractional parts eg $\frac{3}{8}$ means 3 equal parts of 8
- comparing unit fractions by referring to the denominator or diagrams eg $\frac{1}{8}$ is less than $\frac{1}{2}$
- renaming $\frac{2}{2}, \frac{4}{4}, \frac{8}{8}$ as 1
- modelling, comparing and representing fractions with denominators 5, 10 and 100 by extending the knowledge and skills covered above to fifths, tenths and hundredths
- modelling, comparing and representing decimals to two decimal places
- applying an understanding of place value to express whole numbers, tenths and hundredths as decimals
- interpreting decimal notation for tenths and hundredths eg 0.1 is the same as $\frac{1}{10}$
- adding and subtracting decimals with the same number of decimal places (to 2 decimal places)


## Working Mathematically

## Students learn to

- pose questions about a collection of items eg 'Is it possible to show one-eighth of this collection of objects?' ( Questioning)
- explain why $\frac{1}{8}$ is less than $\frac{1}{4}$
eg if the cake is divided among eight people, the slices are smaller than if the cake is shared among four people (Reasoning, Communicating)
- check whether an answer is correct by using an alternative method eg use a number line or calculator to show that $\frac{1}{2}$ is the same as 0.5 and $\frac{5}{10}$ (Reasoning)
- interpret the everyday use of fractions and decimals, such as in advertisements (Reflecting)
- interpret a calculator display in the context of the problem eg 2.6 means $\$ 2.60$ when using money (Applying Strategies, Communicating)
- apply decimal knowledge to record measurements eg $123 \mathrm{~cm}=1.23 \mathrm{~m}$ (Reflecting)
- explain the relationship between fractions and decimals eg $\frac{1}{2}$ is the same as 0.5
(Reasoning, Communicating)
- perform calculations with money (Applying Strategies)


## Background Information

At this Stage, 'commonly used fractions' refers to those with denominators 2,4 and 8 , as well as those with denominators 5,10 and 100 . Students apply their understanding of fractions with denominators 2,4 and 8 to fractions with denominators 5,10 and 100 .

Fractions are used in different ways:

- to describe equal parts of a whole
- to describe equal parts of a collection of objects
- to denote numbers
eg $\frac{1}{2}$ is midway between 0 and 1 on the number line
- as operators related to division
eg dividing a number in half.


## Language

At this Stage it is not intended that students necessarily use the terms 'numerator' and 'denominator'.
'Decimal' is a commonly used contraction of 'decimal fraction'.

In most cases, there are differences in the meaning of fraction and ordinal terms that use the same word eg 'tenth' (fraction) has a different meaning to 'the tenth' (ordinal).

## Stage 2

## Fractions and Decimals - Unit 2

## NS2. 4

Models, compares and represents commonly used fractions and decimals, adds and subtracts decimals to two decimal places, and interprets everyday percentages

## Key Ideas

Find equivalence between halves, quarters and eighths; fifths and tenths; tenths and hundredths
Recognise percentages in everyday situations
Relate a common percentage to a fraction or decimal

## Working Mathematically

## Students learn to

- pose questions about a collection of items eg 'Is it possible to show one-eighth of this collection of objects?' (Questioning)
- check whether an answer is correct by using an alternative method eg use a number line or calculator to show that $\frac{1}{2}$ is the same as 0.5 and $\frac{5}{10}$ (Reasoning)
- interpret the everyday use of fractions, decimals and percentages, such as in advertisements (Reflecting)
- interpret a calculator display in the context of the problem eg 2.6 means $\$ 2.60$ when using money (Applying Strategies, Communicating)
- apply decimal knowledge to record measurements eg $123 \mathrm{~cm}=1.23 \mathrm{~m}$ (Reflecting)
- explain the relationship between fractions and decimals $\operatorname{eg} \frac{1}{2}$ is the same as 0.5
(Reasoning, Communicating)
- round an answer obtained by using a calculator, to one or two decimal places (Applying Strategies)
- use a calculator to create patterns involving decimal numbers eg $1 \div 10,2 \div 10,3 \div 10$ (Applying Strategies)
- perform calculations with money (Applying Strategies)
- modelling, comparing and representing fractions with denominators 5, 10 and 100 by
- extending the knowledge and skills covered above to fifths, tenths and hundredths
- ordering decimals with the same number of decimal places (to 2 decimal places) on a number line
- rounding a number with one or two decimal places to the nearest whole number
- recognising the number pattern formed when decimal numbers are multiplied or divided by 10 or 100
- recognising that the symbol \% means 'percent'
- relating a common percentage to a fraction or decimal eg ' $25 \%$ means 25 out of 100 or 0.25 '
- equating $10 \%$ to $\frac{1}{10}, 25 \%$ to $\frac{1}{4}$ and $50 \%$ to $\frac{1}{2}$


## Background Information

Money is an application of decimals to two decimal places.

At this Stage it is not intended that students necessarily use the terms 'numerator' and 'denominator'.

## Language

The decimal 1.12 is read 'one point one two' and not 'one point twelve'.

The word cent comes from the Latin word 'centum' meaning 'one hundred'. Percent means 'out of one hundred' or 'hundredths'.

Stage 3

## Fractions and Decimals - Unit 1

## NS3.4

Compares, orders and calculates with decimals, simple fractions and simple percentages

## Key Ideas

Model, compare and represent commonly used fractions (those with denominators $2,3,4,5,6,8,10,12$ and 100) Find equivalence between thirds, sixths and twelfths Express a mixed numeral as an improper fraction and vice versa

Multiply and divide decimals by whole numbers in everyday contexts
Add and subtract decimals to three decimal places

## Working Mathematically

## Students learn to

- pose and solve problems involving simple proportions eg 'If a recipe for 8 people requires 3 cups of sugar, how many cups would be needed for 4 people?' (Questioning, Applying Strategies)
- explain or demonstrate why two fractions are or are not equivalent (Reasoning, Reflecting)
- use estimation to check whether an answer is reasonable (Applying Strategies, Reasoning)
- interpret and explain the use of fractions, decimals and percentages in everyday contexts eg $\frac{3}{4} \mathrm{hr}=45 \mathrm{~min}$ (Communicating, Reflecting)
- apply the four operations to money problems (Applying Strategies)
- interpret an improper fraction in an answer (Applying Strategies)
- use a calculator to explore the effect of multiplying or dividing decimal numbers by multiples of ten (Applying Strategies)
- using written, diagram and mental strategies to subtract a unit fraction from any whole number
eg $4-\frac{1}{3}$
- adding and subtracting fractions with the same denominator eg $\frac{5}{6}+\frac{3}{6}$
- expressing thousandths as decimals
- interpreting decimal notation for thousandths
- comparing and ordering decimal numbers with three decimal places
- placing decimal numbers on a number line between 0 and 1
- adding and subtracting decimal numbers with a different number of decimal places
- multiplying and dividing decimal numbers by single digit numbers and by $10,100,1000$


## Background Information

Fractions may be interpreted in different ways depending on the context eg two quarters $\left(\frac{2}{4}\right)$ may be thought of as two equal parts of one whole that has been divided into four equal parts.


Alternatively, two quarters $\left(\frac{2}{4}\right)$ may be thought of as two equal parts of two wholes that have each been divided into quarters.


Students need to interpret a variety of word problems and translate them into mathematical diagrams and/or fraction notation. Fractions have different meanings depending on the context eg show on a diagram $\frac{3}{4}$ of a pizza; four children share three pizzas, draw a diagram to show how much each receives.

## Stage 3

## Fractions and Decimals - Unit 2

## NS3.4

Compares, orders and calculates with decimals, simple fractions and simple percentages

## Key Ideas

Add and subtract simple fractions where one denominator is a multiple of the other
Multiply simple fractions by whole numbers
Calculate unit fractions of a number
Calculate simple percentages of quantities
Apply the four operations to money in real-life situations

## Working Mathematically

## Students learn to

- pose and solve problems involving simple proportions eg 'If a recipe for 8 people requires 3 cups of sugar, how many cups would be needed for 4 people?' (Questioning, Applying Strategies)
- explain or demonstrate why two fractions are or are not equivalent (Reasoning, Reflecting)
- use estimation to check whether an answer is reasonable (Applying Strategies, Reasoning)
- interpret and explain the use of fractions, decimals and percentages in everyday contexts eg $\frac{3}{4} \mathrm{hr}=45 \mathrm{~min}$ (Communicating, Reflecting)
- recall commonly used equivalent fractions eg 75\%, 0.75, $\frac{3}{4}$ (Communicating, Reflecting)
- apply the four operations to money problems (Applying Strategies)
- use mental strategies to convert between percentages and fractions to estimate discounts (Applying Strategies)
- calculate prices following percentage discounts eg a 10\% discount (Applying Strategies)
- explain how $50 \%$ of an amount could be less than $10 \%$ of another amount (Applying Strategies, Reasoning)
- interpret an improper fraction in an answer (Applying Strategies)
- use a calculator to explore and create patterns with fractions and decimals (Applying Strategies)


## Background Information

At this Stage, 'simple fractions' refers to those with denominators $2,3,4,5,6,8,10,12$ and 100 .
Fraction concepts are applied in other areas of mathematics eg Chance, Space and Geometry, and Measurement.
In HSIE, scale is used when reading and interpreting maps.

In music, reading and interpreting note values links with fraction work. Semiquavers, quavers, crotchets, minims and semibreves can be compared using fractions eg a quaver is $\frac{1}{2}$ of a crotchet, and $\frac{1}{4}$ of a minim. Musicians indicate fraction values by tails on the stems of notes or by contrasting open and closed notes.
Time signatures in music appear similar to fractions.

## Language

In Chance, the likelihood of an outcome may be described as, for example 'one in four'.

Students may need assistance with the subtleties of the English language when solving problems eg ' $10 \%$ of $\$ 50$ ' is not the same as ‘ $10 \%$ off $\$ 50$ '.

Stage 4

## Fractions, Decimals and Percentages

## NS4.3

Operates with fractions, decimals, percentages, ratios and rates

## Knowledge and Skills

## Students learn about

## Fractions, Decimals and Percentages

- finding highest common factors and lowest common multiples
- finding equivalent fractions
- reducing a fraction to its lowest equivalent form
- adding and subtracting fractions using written methods
- expressing improper fractions as mixed numerals and vice versa
- adding mixed numerals
- subtracting a fraction from a whole number
eg $3-\frac{2}{3}=2+1-\frac{2}{3}=2 \frac{1}{3}$
- multiplying and dividing fractions and mixed numerals
- adding, subtracting, multiplying and dividing decimals (for multiplication and division, limit operators to two-digits)
- determining the effect of multiplying or dividing by a number less than one
- rounding decimals to a given number of places
- using the notation for recurring (repeating) decimals eg $0.33333 \ldots=0 . \dot{3}, 0.345345345 \ldots=0 . \dot{3} 4 \dot{5}$
- converting fractions to decimals (terminating and recurring) and percentages
- converting terminating decimals to fractions and percentages
- converting percentages to fractions and decimals
- calculating fractions, decimals and percentages of quantities
- increasing and decreasing a quantity by a given percentage
- interpreting and calculating percentages greater than $100 \%$ eg an increase from 6 to 18 is an increase of $200 \%$; $150 \%$ of $\$ 2$ is $\$ 3$
- expressing profit and/or loss as a percentage of cost price or selling price
- ordering fractions, decimals and percentages
- expressing one quantity as a fraction or a percentage of another eg 15 minutes is $\frac{1}{4}$ or $25 \%$ of an hour


## Key Ideas

Perform operations with fractions, decimals and mixed numerals
Use ratios and rates to solve problems

## Working Mathematically

## Students learn to

- explain multiplication of a fraction by a fraction using a diagram to illustrate the process (Reasoning, Communicating)
- explain why division by a fraction is equivalent to multiplication by its reciprocal (Reasoning, Communicating)
- choose the appropriate equivalent form for mental computation eg $10 \%$ of $\$ 40$ is $\frac{1}{10}$ of $\$ 40$ (Applying Strategies)
- recognise and explain incorrect operations with fractions eg explain why $\frac{2}{3}+\frac{1}{4} \neq \frac{3}{7}$
(Applying Strategies, Reasoning, Communicating)
- question the reasonableness of statements in the media that quote fractions, decimals or percentages eg 'the number of children in the average family is 2.3' (Questioning)
- interpret a calculator display in formulating a solution to a problem, by appropriately rounding a decimal (Communicating, Applying Strategies)
- recognise equivalences when calculating eg multiplication by 1.05 will increase a number/quantity by $5 \%$, multiplication by 0.87 will decrease a number/quantity by $13 \%$
(Applying Strategies)
- solve a variety of real-life problems involving fractions, decimals and percentages (Applying Strategies)
- use a number of strategies to solve unfamiliar problems, including:
- using a table
- looking for patterns
- simplifying the problem
- drawing a diagram
- working backwards
- guess and refine
(Applying Strategies, Communicating)
- interpret media and sport reports involving percentages (Communicating)
- evaluate best buys and special offers eg discounts (Applying Strategies)


## Stage 4

## Fractions, Decimals and Percentages (continued)

## Ratio and Rates

- using ratio to compare quantities of the same type
- writing ratios in various forms
eg $\frac{4}{6}, 4: 6,4$ to 6
- $\quad$ simplifying ratios eg $4: 6=2: 3, \frac{1}{2}: 2=1: 4,0.3: 1=3: 10$
- applying the unitary method to ratio problems
- dividing a quantity in a given ratio
- interpreting and calculating ratios that involve more than two numbers
- calculating speed given distance and time
- calculating rates from given information eg 150 kilometres travelled in 2 hours
- interpret descriptions of products that involve fractions, decimals, percentages or ratios eg on labels of packages (Communicating)
- solve a variety of real-life problems involving ratios eg scales on maps, mixes for fuels or concrete, gear ratios (Applying Strategies)
- solve a variety of real-life problems involving rates eg batting and bowling strike rates, telephone rates, speed, fuel consumption (Applying Strategies)


## Background Information

Fraction concepts are applied in other areas of mathematics eg simplifying algebraic expressions, Probability, Trigonometry, and Measurement. Ratio work links with scale drawing, trigonometry and gradient of lines.
In Geography, students calculate percentage change using statistical data, and scale is used when reading and interpreting maps.
In Music, reading and interpreting note values links with fraction work. Semiquavers, quavers, crotchets, minims and semibreves can be compared using fractions eg a quaver is $\frac{1}{2}$ of a crotchet, and $\frac{1}{4}$ of a minim. Musicians indicate fraction values by tails on the stems of notes or by contrasting open and closed notes.
Time signatures in music appear similar to fractions.
In PDHPE there are opportunities for students to apply number skills eg

- when comparing time related to work, leisure and rest, students could express each as a percentage
- assessing the effect of exercise on the body by measuring the increase in pulse rate and body temperature
- calculating the height/weight ratio when analysing body composition
- conducting fitness tests such as allowing 12 minutes for a 1.6 kilometre run.

Work with ratio may be linked with the Golden Rectangle. Many windows are Golden Rectangles, as are some of the buildings in Athens such as the Parthenon. The ratio of the dimensions of the Golden Rectangle was known to the ancient Greeks:
$\frac{\text { length }}{\text { width }}=\frac{\text { length }+ \text { width }}{\text { length }}$
The word fraction comes from the Latin frangere meaning 'to break'. The earliest evidence of fractions can be traced to the Egyptian papyrus of Ahmes (about 1650 BC ). In the seventh century AD the method of writing fractions as we write them now was invented in India, but without the fraction bar (vinculum), which was introduced by the Arabs. Fractions were widely in use by the 12 th century.
The word 'cent' comes from the Latin word 'centum' meaning 'one hundred'. Percent means 'out of one hundred' or 'hundredths'.
One cent and two cent coins were withdrawn by the Australian Government in 1990. Prices can still be expressed in one-cent increments but the final bill is rounded to the nearest five cents. In this context, rounding is different to normal conventions in that totals ending in $3,4,6$, and 7 are rounded to the nearest 5 cents, and totals ending in 8, 9, 1, and 2 are rounded to the nearest 0 cents.

## Language

Students may need assistance with the subtleties of the English language when solving word problems
eg ' $\frac{1}{10}$ of $\$ 50$ ' is not the same as ' $\frac{1}{10}$ off $\$ 50$ '.

Students may wrongly interpret words giving a mathematical instruction (eg estimate, multiply, simplify) to just mean 'get the answer'.

Stage 1

## Chance

## NS1.5

Recognises and describes the element of chance in everyday events

## Key Ideas

Recognise the element of chance in familiar daily activities Use familiar language to describe the element of chance

## Students learn about

- using familiar language to describe chance events eg might, certain, probably, likely, unlikely
- recognising and describing the element of chance in familiar activities eg 'I might play with my friend after school.'
- distinguishing between possible and impossible events
- comparing familiar events and describing them as being more or less likely to happen


## Working Mathematically

## Students learn to

- describe familiar events as being possible or impossible (Communicating)
- describe possible outcomes in everyday situations eg deciding what might occur in a story before the ending of a book (Communicating, Reflecting)
- predict what might occur during the next lesson in class or in the near future eg predict 'How many people might come to your party?'; 'How likely is it to rain if there are no clouds in the sky?' (Reflecting)


## Background Information

Students should be encouraged to recognise that, because of the element of chance, their predictions will not always be proven true.

When discussing certainty, there are two extremes: events that are certain to happen and those that are certain not to happen. Words such as 'might', 'may', 'possible' are between these two extremes.

## Language

The meaning of 'uncertain' is 'not certain' - it does not mean 'impossible'.

Stage 2

## Chance

## NS2.5

Describes and compares chance events in social and experimental contexts

## Key Ideas

Explore all possible outcomes in a simple chance situation
Conduct simple chance experiments
Collect data and compare likelihood of events in different contexts

## Working Mathematically

## Students learn to

- discuss the 'fairness' of simple games involving chance (Communicating)
- compare the likelihood of outcomes in a simple chance experiment eg from a collection of 27 red, 10 blue and 13 yellow marbles, name red as being the colour most likely to be drawn out (Reasoning)
- apply an understanding of equally likely outcomes in situations involving random generators such as dice, coins and spinners (Reflecting)
- make statements that acknowledge 'randomness' in a situation eg 'the spinner could stop on any colour' (Communicating, Reflecting)
- explain the differences between expected results and actual results in a simple chance experiment (Communicating, Reflecting)


## Background Information

When a fair coin is tossed, theoretically there is an equal chance of a head or tail. If the coin is tossed and there are five heads in a row there is still an equal chance of a head or tail on the next toss, since each toss is an independent event.

Stage 3
Chance

## NS3.5

Orders the likelihood of simple events on a number line from zero to one

## Knowledge and Skills

## Students learn about

- using data to order chance events from least likely to most likely eg roll two dice twenty times and order the results according to how many times each total is obtained
- ordering commonly used 'chance words' on a number line between zero (impossible) and one (certain) eg 'equal chance' would be placed at 0.5
- using knowledge of equivalent fractions and percentages to assign a numerical value to the likelihood of a simple event occurring eg there is a five in ten, $\frac{5}{10}, 50 \%$ or one in two chance of this happening
- describing the likelihood of events as being more or less than a half ( $50 \%$ or 0.5 ) and ordering the events on a number line
- using samples to make predictions about a larger 'population' from which the sample comes eg predicting the proportion of cubes of each colour in a bag after taking out a sample of the cubes


## Key Ideas

Assign numerical values to the likelihood of simple events occurring
Order the likelihood of simple events on a number line from 0 to 1

## Working Mathematically

## Students learn to

- predict and discuss whether everyday events are more or less likely to occur or whether they have an equal chance of occurring
(Applying Strategies, Communicating)
- assign numerical values to the likelihood of simple events occurring in real-life contexts
eg 'My football team has a fifty-fifty chance of winning the game.' (Applying Strategies, Reflecting)
- describe the likelihood of an event occurring as being more or less than half (Communicating, Reflecting)
- question whether their prediction about a larger population, from which a sample comes, would be the same if a different sample was used eg 'Would the results be the same if a different class was surveyed?' (Questioning, Reflecting)
- design a spinner or label a die so that a particular outcome is more likely than another (Applying Strategies)


## Background Information

Students will need some prior experience ordering decimal fractions (tenths) on a number line from zero to one.
There is a need for students to represent all possible outcomes for a single stage experiment in an organised way eg tables, grids, tree diagrams.

Chance events can be ordered on a scale from zero to one. A chance of zero describes an event that is impossible. A chance of one describes an event that is certain. Therefore, events with an equal chance of occurring can be described as having a chance of 0.5 . Other expressions of chance fall between zero and one eg 'unlikely' will take a numerical value somewhere between 0 and 0.5 .

Stage 4

## Probability

## NS4. 4

Solves probability problems involving simple events

## Key Ideas

Determine the probability of simple events
Solve simple probability problems
Recognise complementary events

## Knowledge and Skills

## Students learn about

- listing all possible outcomes of a simple event
- using the term 'sample space' to denote all possible outcomes eg for tossing a fair die, the sample space is 1 , 2, 3, 4, 5, 6
- assigning probabilities to simple events by reasoning about equally likely outcomes eg the probability of a 5 resulting from the throw of a fair die is $\frac{1}{6}$
- expressing the probability of a particular outcome as a fraction between 0 and 1
- assigning a probability of zero to events that are impossible and a probability of one to events that are certain
- recognising that the sum of the probabilities of all possible outcomes of a simple event is 1
- identifying the complement of an event eg 'The complement of drawing a red card from a deck of cards is drawing a black card.'
- finding the probability of a complementary event


## Working Mathematically

## Students learn to

- solve simple probability problems arising in games (Applying Strategies)
- use language associated with chance events appropriately (Communicating)
- evaluate media statements involving probability (Applying Strategies, Communicating)
- interpret and use probabilities expressed as percentages or decimals (Applying Strategies, Communicating)
- explain the meaning of a probability of $0, \frac{1}{2}$ and 1 in a given situation (Communicating, Reasoning)


## Background Information

A simple event is an event in which each possible outcome is equally likely eg tossing a fair die.

## Patterns and Algebra

The Patterns and Algebra strand has been incorporated into the primary curriculum to demonstrate the importance of early number learning in the development of algebraic thinking. This strand emphasises number patterns and number relationships leading to an investigation of the way that one quantity changes relative to another.

The Patterns and Algebra strand extends from Early Stage 1 to Stage 5.3. In the early Stages students explore number and pre-algebra concepts by pattern making, and discussing, generalising and recording their observations. Separating these concepts into a distinct strand is intended to demonstrate the connections between these early understandings and the algebra concepts that follow. The Patterns and Algebra strand links with the Number strand and it is recommended that it be taught in conjunction with the development of number concepts.

One important aspect of algebraic thinking is the development of students' abilities to replicate, complete, continue, describe, generalise and create repeating patterns and number patterns that increase or decrease. These number patterns can be formed using rhythmic or skip counting.

Repeating patterns can be created using sounds, actions, shapes, objects, stamps, pictures and other materials. Children could be encouraged to create a wide variety of such patterns and then to describe and label them using numbers. Repeating patterns can be described using numbers that indicate the number of elements that repeat. For example, $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{A}, \mathrm{B}, \mathrm{C}, \ldots$ has three elements that repeat and is referred to as a 'three' pattern; $\mathrm{D}, \mathrm{0}, \mathrm{0}, \mathrm{D}, 0,0$, ... is also a three pattern because there is a sequence of three repeating elements.

Another important aspect of algebraic thinking is the ability to recognise and use number relationships and to be able to make generalisations about number relationships. From Early Stage 1, children should be encouraged to describe number relationships and to make generalisations when appropriate. In addition, finding unknowns or missing elements in number sentences needs to be addressed from an early Stage. This is associated with the concept of equality and the need to develop an understanding that the equals sign also means 'is the same as'.

This section presents the outcomes, key ideas, knowledge and skills, and Working Mathematically statements from Early Stage 1 to Stage 3 in one substrand. The Stage 4 content is presented in the topics: Number Patterns, Algebraic Techniques, and Linear Relationships.

## Early Stage 1

## Patterns and Algebra

## PAES1. 1

Recognises, describes, creates and continues repeating patterns and number patterns that increase or decrease

## Knowledge and Skills

## Students learn about Repeating Patterns and Number Patterns

- recognising, copying and continuing repeating patterns using sounds and/or actions
- recognising, copying, continuing and creating repeating patterns using shapes, objects or pictures
eg $\downarrow, \nabla, \downarrow, \nabla, \downarrow, \nabla \ldots$
- describing a repeating pattern made from shapes by referring to distinguishing features
eg 'I have made my pattern from squares. The colours repeat. They go red, blue, red, blue, ...'
- describing a repeating pattern in terms of a 'number' pattern
eg $\bullet, \mathrm{O}, \stackrel{\mathrm{O}, \star, \mathrm{O}, \ldots \text { is a 'two' pattern }}{ }$
$\Delta, \nabla, \mathrm{O}, \Delta, \nabla, \mathrm{O}, \ldots$ is a 'three' pattern $\mathrm{B}, \mathrm{B}, \mathrm{X}, \mathrm{B}, \mathrm{B}, \mathrm{X}, \ldots$ is a 'three' pattern
- recognising, copying and continuing simple number patterns that increase or decrease
eg $1,2,3,4, \ldots$

$$
20,19,18,17, \ldots
$$

$2,4,6,8, \ldots$

## Number Relationships

- using the term 'is the same as' to express equality of groups


## Key Ideas

Recognise, describe, create and continue repeating patterns Continue simple number patterns that increase or decrease Use the term 'is the same as' to describe equality of groups

Working Mathematically

## Students learn to

- ask questions about how repeating patterns are made and how they can be copied or continued (Questioning)
- check solutions to continuing a pattern by repeating the process (Applying Strategies, Reasoning)
- record patterns created by using the process of repeatedly adding the same number on a calculator (Communicating)
- create repeating patterns with the same 'number' pattern eg A, B, B, A, B, B, ... is a 'three' pattern and so is o, $\Delta, \Delta$, o, $\Delta, \Delta, \ldots$
(Communicating, Applying Strategies)
- recognise when an error occurs in a pattern and explain what is wrong (Applying Strategies, Communicating, Reasoning)
- make connections between counting and repeating patterns (Reflecting)
- create or continue a repeating pattern using simple computer graphics (Applying Strategies)
- determine whether two groups have the same number of objects and describe the equality
eg 'The number of objects here is the same as the number there.' (Applying Strategies, Communicating)


## Background Information

Early number learning is important to the development of algebraic thinking in later Stages.

## Repeating Patterns and Number Patterns

At this Stage, repeating patterns can be created using sounds, actions, shapes, objects, stamps, pictures and other materials. Describing and labelling these patterns using numbers is important.

Repeating patterns are described using numbers that indicate, the number of elements that repeat eg 'A, B, C, A, B, C, ...' has three elements that repeat and is referred to as a 'three' pattern.
Number Relationships
At this Stage, forming groups of objects that have the same number of elements helps to develop the concept of equality.

## Stage 1

## Patterns and Algebra

## PAS1. 1

Creates, represents and continues a variety of number patterns, supplies missing elements in a pattern and builds number relationships

## Key Ideas

Create, represent and continue a variety of number patterns and supply missing elements
Use the equals sign to record equivalent number relationships
Build number relationships by relating addition and subtraction facts to at least 20
Make generalisations about number relationships

## Knowledge and Skills

## Students learn about Number Patterns

- identifying and describing patterns when counting forwards or backwards by ones, twos, fives, or tens
- continuing, creating and describing number patterns that increase or decrease
- representing number patterns on a number line or hundreds chart
- determining a missing element in a number pattern eg 3, 7, 11, ?, 19, 23, 27
- modelling and describing odd and even numbers using counters paired in two rows


## Number Relationships

- using the equals sign to record equivalent number relationships and to mean 'is the same as' rather than as an indication to perform an operation
eg $5+2=4+3$
- building addition facts to at least 20 by recognising patterns or applying the commutative property eg $4+5=5+4$
- relating addition and subtraction facts for numbers to at least 20 eg $5+3=8$; so $8-3=5$ and $8-5=3$
- modelling and recording patterns for individual numbers by making all possible whole number combinations
eg $\quad 0+4=4$
$1+3=4$
$2+2=4$
$3+1=4$
$4+0=4$
- finding and making generalisations about number relationships eg adding zero does not change the number, as in $6+0=6$


## Working Mathematically

## Students learn to

- pose and solve problems based on number patterns ( Questioning, Applying Strategies)
- ask questions about how number patterns are made and how they can be copied or continued (Questioning)
- describe how the missing element in a number pattern was determined (Communicating, Reflecting)
- check solutions to missing elements in patterns by repeating the process (Reasoning)
- generate number patterns using the process of repeatedly adding the same number on a calculator (Communicating)
- represent number patterns using diagrams, words or symbols (Communicating)
- describe what has been learnt from creating patterns, making connections with addition and related subtraction facts (Reflecting)
- recognise patterns created by adding combinations of odd and even numbers eg odd + odd $=$ even, odd + even $=$ odd (Reflecting)
- check number sentences to determine if they are true or false, and if false, describe why eg Is $7+5=8+5$ true? If not, why not?
(Communicating, Reasoning)


## Background Information

## Number Patterns

At this Stage, students further explore number patterns that increase or decrease. Patterns could now include any patterns observed on a hundreds chart and these might go beyond patterns created by counting in ones, twos, fives or tens. This links closely with the development of Whole Numbers and Multiplication and Division.

## Number Relationships

At this Stage, describing number relationships and making generalisations should be encouraged when appropriate. The concept of equality and the understanding that the equals sign also means 'is the same as' is important.

## Stage 2

## Patterns and Algebra

## PAS2.1

Generates, describes and records number patterns using a variety of strategies and completes simple number sentences by calculating missing values

## Key Ideas

Generate, describe and record number patterns using a variety of strategies
Build number relationships by relating multiplication and division facts to at least $10 \times 10$
Complete simple number sentences by calculating the value of a missing number

## Knowledge and Skills

## Students learn about

## Number Patterns

- identifying and describing patterns when counting forwards or backwards by threes, fours, sixes, sevens, eights or nines
- creating, with materials or a calculator, a variety of patterns using whole numbers, fractions or decimals
eg $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \frac{5}{4}, \frac{6}{4}, \ldots$

$$
2.2,2.0,1.8,1.6, \ldots
$$

- finding a higher term in a number pattern given the first five terms eg determine the $10^{\text {th }}$ term given a number pattern beginning with $4,8,12,16,20, \ldots$
- describing a simple number pattern in words


## Number Relationships

- using the equals sign to record equivalent number relationships and to mean 'is the same as' rather than as an indication to perform an operation eg $4 \times 3=6 \times 2$
- building the multiplication facts to at least $10 \times 10$ by recognising and describing patterns and applying the commutative property eg $6 \times 4=4 \times 6$
- forming arrays using materials to demonstrate multiplication patterns and relationships

$$
\begin{aligned}
\text { eg } & 3 \times 5=15 \quad \\
& \bullet \bullet \bullet \cdot \\
& \bullet \bullet \bullet
\end{aligned}
$$

- relating multiplication and division facts eg $6 \times 4=24$; so $24 \div 4=6$ and $24 \div 6=4$
- applying the associative property of addition and multiplication to aid mental computation eg $2+3+8=2+8+3,2 \times 3 \times 5=2 \times 5 \times 3$
- completing number sentences involving one operation by calculating missing values

$$
\text { eg find } \square \text { so that } 5+\square=13 \text {, }
$$

$$
\text { find } \square \text { so that } 28=\square \times 7
$$

- transforming a division calculation into a multiplication problem eg find $\square$ so that $30 \div 6=\square$ becomes find $\square$ so that $\square \times 6=30$.


## Working Mathematically

## Students learn to

- pose problems based on number patterns (Questioning)
- solve a variety of problems using problem-solving strategies, including:
- trial and error
- drawing a diagram
- working backwards
- looking for patterns
- using a table (Applying Strategies, Communicating)
- ask questions about how number patterns have been created and how they can be continued (Questioning)
- generate a variety of number patterns that increase or decrease and record them in more than one way (Applying Strategies, Communicating)
- generate number patterns using the process of repeatedly adding the same number on a calculator (Communicating)
- model and then record number patterns using diagrams, words or symbols (Communicating)
- check solutions to missing elements in patterns by repeating the process (Reasoning)
- play 'guess my rule' games eg $1,4,7$ : what is the rule? (Applying Strategies)
- describe what has been learnt from creating patterns, making connections with addition facts and multiplication facts (Communicating, Reflecting)
- explain the relationship between multiplication facts eg explain how the 3 and 6 times tables are related (Reflecting)
- make generalisations about numbers and number relationships eg 'It doesn't matter what order you multiply two numbers because the answer is always the same.' (Reflecting)
- check number sentences to determine if they are true or false, and, if false, explain why (Applying Strategies, Reasoning)
- justify a solution to a number sentence (Reasoning)
- use inverse operations to complete number sentences (Applying Strategies)
- describe strategies for completing simple number sentences (Communicating)

Stage 3

## Patterns and Algebra

## PAS3.1a

Records, analyses and describes geometric and number patterns that involve one operation using tables and words

## Key Ideas

Build simple geometric patterns involving multiples Complete a table of values for geometric and number patterns Describe a pattern in words in more than one way

## Knowledge and Skills

## Students learn about

- working through a process of building a simple geometric pattern involving multiples, completing a table of values, and describing the pattern in words. This process includes the following steps:
- building a simple geometric pattern using materials
eg $\Delta, \Delta \Delta, \Delta \Delta \Delta, \Delta \Delta \Delta \Delta, \ldots$
- completing a table of values for the geometric pattern
eg

| Number of Triangles | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of Sides | 3 | 6 | 9 | 12 | - | - |

- describing the number pattern in a variety of ways and recording descriptions using words eg 'It looks like the three times tables.'
- determining a rule to describe the pattern from the table eg 'You multiply the top number by three to get the bottom number.'
- using the rule to calculate the corresponding value for a larger number
- working through a process of identifying a simple number pattern involving only one operation, completing a table of values, and describing the pattern in words. This process includes the following steps:
- completing a table of values for a number pattern involving one operation (including patterns that decrease)
eg

| First Number | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Second Number | 4 | 5 | 6 | 7 | - | - |

- describing the pattern in a variety of ways and recording descriptions using words
- determining a rule to describe the pattern from the table
- using the rule to calculate the corresponding value for a larger number


## Working Mathematically

## Students learn to

- ask questions about how number patterns have been created and how they can be continued (Questioning)
- interpret sentences written by peers and teachers that accurately describe geometric and number patterns (Applying Strategies)
- identify patterns in data displayed in a spreadsheet (Applying Strategies)
- generate a variety of number patterns that increase or decrease and record in more than one way (Applying Strategies, Communicating)
- model and then record number patterns using materials, diagrams, words or symbols (Applying Strategies)
- use a number of strategies to solve unfamiliar problems, including:
- trial and error
- drawing a diagram
- working backwards
- looking for patterns
- using a table
(Applying Strategies, Communicating)
- check solutions to missing elements in patterns by repeating the process (Reasoning)
- describe what has been learnt from creating patterns, making connections with number facts and number properties (Communicating, Reflecting)
- make generalisations about numbers and number relationships eg 'If you add a number and then subtract the same number, the result is the number you started with.' (Reflecting)
- play ‘guess my rule’ games (Applying Strategies)
- describe and justify the choice of a particular rule for the values in a table (Communicating, Reasoning)


## Background Information

This topic involves algebra without symbols. Symbols should not be introduced until the students have had considerable experience describing patterns in their own words.

Students should be given opportunities to discover and create patterns and to describe, in their own words, relationships contained in those patterns.

## Language

At this Stage, students should be encouraged to use their own words to describe number patterns. Patterns can usually be described in more than one way and it is important for students to hear how other students describe the same pattern.

Students' descriptions of number patterns can then become more sophisticated as they experience a variety of ways of describing the same pattern. The teacher could begin to model the use of more appropriate mathematical language to encourage this development.

Stage 3

## Patterns and Algebra

## PAS3.1b

Constructs, verifies and completes number sentences involving the four operations with a variety of numbers

## Knowledge and Skills

## Students learn about

- completing number sentences that involve more than one operation by calculating missing values eg Find $\square$ so that $5+\square=12-4$
- completing number sentences involving fractions or decimals eg Find $\square$ so that $7 \times \square=7.7$
- constructing a number sentence to match a problem that is presented in words and requires finding an unknown eg 'I am thinking of a number so that when I double it and add 5 the answer is 13 . What is the number?'
- checking solutions to number sentences by substituting the solution into the original question
- identifying and using inverse operations to assist with the solution of number sentences eg Find $\square$ so that $125 \div 5=\square$ becomes find $\square$ so that $\square \times 5=125$.


## Key Ideas

Construct, verify and complete number sentences involving the four operations with a variety of numbers

## Working Mathematically

## Students learn to

- describe strategies for completing simple number sentences and justify solutions (Communicating)
- describe how inverse operations can be used to solve a number sentence (Applying Strategies, Communicating)


## Background Information

Students will typically use trial-and-error methods to find solutions to number sentences. They need to be encouraged to work backwards and to describe the processes using inverse operations. The inclusion of sentences that do not have whole number solutions will aid this process.

Stage 4

## Algebraic Techniques

## PAS4. 1

Uses letters to represent numbers and translates between words and algebraic symbols

## Knowledge and Skills

## Students learn about

- using letters (pronumerals) to represent numbers and developing the notion that a letter is used to represent a variable
- using concrete materials such as cups and counters to model:
- expressions that involve a variable and a variable plus a constant eg $a, a+1$
- expressions that involve a variable multiplied by a constant eg $2 a, 3 a$
- sums and products eg $2 a+1,2(a+1)$
- equivalent expressions such as

$$
x+x+y+y+y=2 x+2 y+y=2(x+y)+y
$$

- and to assist with simplifying expressions, such as

$$
\begin{aligned}
(a+2)+(2 a+3) & =(a+2 a)+(2+3) \\
& =3 a+5
\end{aligned}
$$

- recognising and using equivalent algebraic expressions eg $\quad y+y+y+y=4 y$

$$
w \times w=w^{2}
$$

$$
a \times b=a b
$$

$$
a \div b=\frac{a}{b}
$$

- translating between words and algebraic symbols and between algebraic symbols and words


## Key Ideas

Use letters to represent numbers
Translate between words and algebraic symbols and between algebraic symbols and words
Recognise and use simple equivalent algebraic expressions

## Working Mathematically

## Students learn to

- generate a variety of equivalent expressions that represent a particular situation or problem (Applying Strategies)
- describe relationships between the algebraic symbol system and number properties (Reflecting, Communicating)
- link algebra with generalised arithmetic eg for the commutative property, determine that $a+b=b+a$ (Reflecting)
- determine equivalence of algebraic expressions by substituting a given number for the letter (Applying Strategies, Reasoning)


## Background Information

To gain an understanding of algebra, students must be introduced to the concepts of patterns, relationships, variables, expressions, unknowns, equations and graphs in a wide variety of contexts. For each successive context, these ideas need to be redeveloped. Students need gradual exposure to abstract ideas as they begin to relate algebraic terms to real situations.
It is important to develop an understanding of the use of letters (pronumerals) as algebraic symbols for variable numbers of objects rather than for the objects themselves. The practice of using the first letter of the name of an object as a symbol for the number of such objects (or still worse as a symbol for the object) can lead to misconceptions and should be avoided, especially in the early Stages.

## Introducing Letters as Algebraic Symbols

The recommended approach is to spend time over the conventions for using algebraic symbols for first-degree expressions and to situate the translation of generalisations from words to symbols as an application of students’ knowledge of the symbol system rather than as an introduction to the symbol system.

Considerable time needs to be spent manipulating concrete materials, such as cups and counters, to develop a good understanding of the notion of a variable and to establish the equivalence of expressions.
The recommended steps for moving into symbolic algebra are:

- the variable notion, associating letters with a variety of variables
- symbolism for a variable plus a constant
- symbolism for a variable times a constant
- symbolism for sums and products.

When evaluating expressions, there must be an explicit direction to replace the letter by a number to ensure full understanding of notation occurs.
Thus if $a=6, a+a=6+6$ but $2 a=2 \times 6$ and not 26 .
It is suggested that the introduction of the symbol system precede the Number Patterns topic for Stage 4, since this topic presumes students are able to manipulate algebraic symbols and will use them to generalise patterns.

## Stage 4

## Number Patterns

## PAS4. 2

Creates, records, analyses and generalises number patterns using words and algebraic symbols in a variety of ways

## Key Ideas

Create, record and describe number patterns using words Use algebraic symbols to translate descriptions of number patterns
Represent number pattern relationships as points on a grid

## Working Mathematically

## Students learn to

- ask questions about how number patterns have been created and how they can be continued (Questioning)
- generate a variety of number patterns that increase or decrease and record them in more than one way (Applying Strategies, Communicating)
- model and then record number patterns using diagrams, words and algebraic symbols (Communicating)
- check pattern descriptions by substituting further values (Reasoning)
- describe the pattern formed by plotting points from a table and suggest another set of points that might form the same pattern
(Communicating, Reasoning)
- describe what has been learnt from creating patterns, making connections with number facts and number properties (Reflecting)
- play 'guess my rule’ games, describing the rule in words and algebraic symbols where appropriate (Applying Strategies, Communicating)
- represent and apply patterns and relationships in algebraic forms (Applying Strategies, Communicating)
- explain why a particular relationship or rule for a given pattern is better than another (Reasoning, Communicating)
- distinguish between graphs that represent an increasing number pattern and those that represent a decreasing number pattern (Communicating)
- determine whether a particular number pattern can be described using algebraic symbols (Applying Strategies, Communicating)
- determining a rule in words to describe the pattern from the table: this needs to be expressed in function form relating the top-row and bottom-row terms in the table
- describing the rule in words, replacing the varying number by an algebraic symbol
- using algebraic symbols to create an equation that describes the pattern
- creating more than one equation to describe the pattern
- using the rule to calculate the corresponding value for a larger number
- using a process that consists of identifying a number pattern (including decreasing patterns), completing a table of values, describing the pattern in words and algebraic symbols, and representing the relationship on a graph:
- completing a table of values for the number pattern

eg | $a$ | 1 | 2 | 3 | 4 | 5 | 10 | 100 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $b$ | 4 | 7 | 10 | 13 | - | - | - |

Stage 4

## Number Patterns (continued)

- describing the pattern in a variety of ways and recording descriptions using words
- representing the values from the table on a number grid and describing the pattern formed by the points on the graph
- determining a rule in words to describe the pattern from the table - this needs to be expressed in function form relating the top-row and bottom-row terms in the table
- describing the rule in words, replacing the varying number by an algebraic symbol
- using algebraic symbols to create an equation that describes the pattern
- creating more than one equation to describe the pattern
- using the rule to calculate the corresponding value for a larger number


## Background Information

In completing tables, intermediate stages should be encouraged.
Consider the following example of the line of squares that is presented in the 'learn about' statements:
1 modelling geometric patterns using materials such as matchsticks to form squares


2 forming and completing a table of values for the geometric pattern

eg | Number of squares | 1 | 2 | 3 | 4 | 5 | 10 | 100 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of matchsticks | 4 | 7 | 10 | 13 | - | - | - |

It may help students to develop the table as follows:
(i) starting from one match

| Number of <br> squares | 1 | 2 | 3 |
| :--- | :--- | :---: | :---: |
| Number of <br> matches | $1+3=4$ | $1+3+3=7$ | $1+3+3+3=10$ |
| or | $1+3 \times 1=4$ | $1+3 \times 2=7$ | $1+3 \times 3=10$ |
| or | $1+1 \times 3=4$ | $1+2 \times 3=7$ | $1+3 \times 3=10$ |
| or | $\ldots$ |  |  |

or,
(ii) starting from one square

| Number of <br> squares | 1 | 2 | 3 |
| :--- | :---: | :---: | :---: |
| Number of <br> matches | 4 | $4+3 \times 1=7$ | $4+3 \times 2=10$ |

Students recognise relationships in the table of values and extend the table to include cases that would be impractical to build, basing their calculations on their own verbal descriptions of the pattern eg for 102 squares, method (i) would lead to $1+3 \times 102=307$ and method (ii) would lead to $4+3 \times 101=307$.
Similarly, number patterns may be used as sources for verbal generalisations. Emphasis should be given to encouraging students to describe how they can obtain one term from earlier terms.
For example, in the number pattern $1,3,5,7,9, \ldots$ 'you keep adding two to get the next number'

| 1 | $1+2$ | $1+2+2$ | $1+2+2+2$ |
| :---: | :---: | :---: | :---: |
| or 1 | $1+2 \times 1$ | $1+2 \times 2$ | $1+2 \times 3$ |
| or 1 | $\ldots$ |  |  |

Students could build the pattern using concrete materials or represent it using diagrams.
More than one aspect of a geometric pattern may be considered eg perimeter, area, number of corners.
The number plane is introduced in Linear Relationships (PAS4.5). Students could be introduced to the early ideas in that topic before graphing points in this topic.

## Stage 4

## Algebraic Techniques

## PAS4. 3

Uses the algebraic symbol system to simplify, expand and factorise simple algebraic expressions

## Key Ideas

Use the algebraic symbol system to simplify, expand and factorise simple algebraic expressions
Substitute into algebraic expressions

## Knowledge and Skills

## Students learn about

- recognising like terms and adding and subtracting like terms to simplify algebraic expressions eg $2 n+4 m+n=4 m+3 n$
- recognising the role of grouping symbols and the different meanings of expressions, such as
$2 a+1$ and $2(a+1)$
- simplifying algebraic expressions that involve multiplication and division
eg $12 a \div 3$

$$
4 x \times 3
$$

$2 a b \times 3 a$

- simplifying expressions that involve simple algebraic fractions
eg $\frac{a}{2}+\frac{a}{3}$

$$
\frac{2 x}{5}-\frac{x}{3}
$$

- expanding algebraic expressions by removing grouping symbols (the distributive property)

$$
\text { eg } \begin{aligned}
3(a+2) & =3 a+6 \\
-5(x+2) & =-5 x-10 \\
a(a+b) & =a^{2}+a b
\end{aligned}
$$

- factorising a single term eg $6 a b=3 \times 2 \times a \times b$
- factorising algebraic expressions by finding a common factor
eg $\quad 6 a+12=6(a+2)$

$$
x^{2}-5 x=x(x-5)
$$

$$
5 a b+10 a=5 a(b+2)
$$

$$
-4 t-12=-4(t+3)
$$

- distinguishing between algebraic expressions where letters are used as variables, and equations, where letters are used as unknowns
- substituting into algebraic expressions
- generating a number pattern from an algebraic expression

eg | $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 10 | 100 |  |
| :--- | :--- | :--- | :--- | :--- | :---: | :---: | :---: | :---: | :---: |
|  | $x+3$ | 4 | 5 | 6 | - | - | - | - | - |

- replacing written statements describing patterns with equations written in algebraic symbols
eg 'you add five to the first number to get the second number' could be replaced with ' $y=x+5$ '
- translating from everyday language to algebraic language and from algebraic language to everyday language


## Working Mathematically

## Students learn to

- generate a variety of equivalent expressions that represent a particular situation or problem (Applying Strategies)
- determine and justify whether a simplified expression is correct by substituting numbers for letters (Applying Strategies, Reasoning)
- check expansions and factorisations by performing the reverse process (Reasoning)
- interpret statements involving algebraic symbols in other contexts eg creating and formatting spreadsheets (Communicating)
- explain why a particular algebraic expansion or factorisation is incorrect (Reasoning, Communicating)
- determine whether a particular pattern can be described using algebraic symbols
(Applying Strategies, Communicating)

Stage 4

## Algebraic Techniques

## PAS4.4

Uses algebraic techniques to solve linear equations and simple inequalities

## Key Ideas

Solve linear equations and word problems using algebra Solve simple inequalities

## Knowledge and Skills

## Students learn about

- solving simple linear equations using concrete materials, such as the balance model or cups and counters, stressing the notion of doing the same thing to both sides of an equation
- solving linear equations using strategies such as guess, check and improve, and backtracking (reverse flow charts)
- solving equations using algebraic methods that involve up to and including three steps in the solution process and have solutions that are not necessarily whole numbers

$$
\text { eg } \quad \begin{aligned}
3 x+4 & =13 \\
5(a+3) & =14 \\
\frac{3 t-2}{5} & =6 \\
3 g-5 & =g+7
\end{aligned}
$$

- checking solutions to equations by substituting
- translating a word problem into an equation, solving the equation and translating the solution into an answer to the problem
- solving equations arising from substitution into formulae eg given $P=2 l+2 b$ and $P=20, l=6$, solve for $b$
- finding a range of values that satisfy an inequality using strategies such as 'guess and check'
- solving simple inequalities such as

$$
\begin{aligned}
& 6 a \leq 18 \\
& 5 y<14 \\
& \frac{t}{5} \geq-2
\end{aligned}
$$

- representing solutions to simple inequalities on the number line


## Working Mathematically

## Students learn to

- compare and contrast different methods to solve a range of linear equations (Reasoning)
- create equations to solve a variety of problems, clearly stating the meaning of introduced letters as 'the number of ...', and verify solutions
(Applying Strategies, Reasoning)
- use algebraic techniques as a tool for problem solving (Applying Strategies)
- construct formulae for finding areas of common geometric figures eg area of a triangle (Applying Strategies)
- determine equations that have a given solution eg find equations that have the solution $x=5$ (Applying Strategies)
- substitute into formulae used in other strands of the syllabus or in other key learning areas and interpret the solutions (Applying Strategies, Communicating)
eg

$$
\begin{aligned}
c^{2} & =a^{2}+b^{2} \\
S & =\frac{D}{T} \\
C & =\frac{5}{9}(F-32)
\end{aligned}
$$

- describe the process of solving simple inequalities and justifying solutions (Communicating, Reasoning)


## Background Information

Five models have been proposed to assist students with the solving of simple equations.
Model 1 uses a two-pan balance and objects such as coins or centicubes. A light paper wrapping can hide a 'mystery number' of objects without distorting the balance's message of equality.
Model 2 uses small objects (all the same) with some hidden in containers to produce the 'unknowns' or 'mystery numbers'.
eg place the same number of small objects in a number of paper cups and cover them with another cup. Form an equation using the cups and then remove objects in equal amounts from each side of a marked equals sign.
Model 3 uses one-to-one matching of terms on each side of the equation.
eg $\quad 3 x+1=2 x+3$
$x+x+x+1=x+x+2+1$
giving $x=2$ through one-to-one matching.

Model 4 uses a substitution approach. By trial and error a value is found for the unknown that produces equality for the values of the two expressions on either side of the equation (this highlights the variable concept).
Simple equations can usually be solved using arithmetic methods. Students need to solve equations where the solutions are not whole numbers and that require the use of algebraic methods.
Model 5 uses backtracking or a reverse flow chart to unpack the operations and find the solution. This model only works for equations with all letters on the same side.
eg $\quad 3 d+5=17$
$17-5 \rightarrow \square \div 3 \rightarrow$
$17-5 \rightarrow 12 \div 3 \rightarrow 4$
$\therefore d=4$

## Stage 4

## Linear Relationships

## PAS4.5

Graphs and interprets linear relationships on the number plane

## Knowledge and Skills

## Students learn about

- interpreting the number plane formed from the intersection of a horizontal $x$-axis and vertical $y$-axis and recognising similarities and differences between points located in each of the four quadrants
- identifying the point of intersection of the two axes as the origin, having coordinates $(0,0)$
- reading, plotting and naming ordered pairs on the number plane including those with values that are not whole numbers
- graphing points on the number plane from a table of values, using an appropriate scale
- extending the line joining a set of points to show that there is an infinite number of ordered pairs that satisfy a given linear relationship
- interpreting the meaning of the continuous line joining the points that satisfy a given number pattern
- reading values from the graph of a linear relationship to demonstrate that there are many points on the line
- deriving a rule for a set of points that has been graphed on a number plane by forming a table of values or otherwise
- forming a table of values for a linear relationship by substituting a set of appropriate values for either of the letters and graphing the number pairs on the number plane eg given $y=3 x+1$, forming a table of values using $x=0,1$ and 2 and then graphing the number pairs on a number plane with appropriate scale
- graphing more than one line on the same set of axes and comparing the graphs to determine similarities and differences eg parallel, passing through the same point
- graphing two intersecting lines on the same set of axes and reading off the point of intersection


## Key Ideas

Interpret the number plane and locate ordered pairs Graph and interpret linear relationships created from simple number patterns and equations

## Working Mathematically

## Students learn to

- relate the location of points on a number plane to maps, plans, street directories and theatre seating and note the different recording conventions eg $15^{\circ} \mathrm{E}$
(Communicating, Reflecting)
- compare similarities and differences between sets of linear relationships (Reasoning)

$$
\text { eg } \begin{array}{llll}
y & =3 x, & y=3 x+2, & y=3 x-2 \\
y & =x, & y=2 x, & y=3 x \\
y & =-x, & y=x &
\end{array}
$$

- sort and classify equations of linear relationships into groups to demonstrate similarities and differences (Reasoning)
- question whether a particular equation will have a similar graph to another equation and graph the line to check ( Questioning, Applying Strategies, Reasoning)
- recognise and explain that not all patterns form a linear relationship (Reasoning)
- determine and explain differences between equations that represent linear relationships and those that represent non-linear relationships
(Applying Strategies, Reasoning)
- explain the significance of the point of intersection of two lines in relation to it being a solution of each equation (Applying Strategies, Reasoning)
- question if the graphs of all linear relationships that have a negative $x$ term will decrease (Questioning)
- reason and describe which term affects the slope of a graph, making it either increasing or decreasing (Reasoning, Communicating)
- use a graphics calculator and spreadsheet software to graph and compare a range of linear relationships (Applying Strategies, Communicating)


## Background Information

In this topic, the notion of locating position that was established in Stage 3 in the Space and Geometry strand is further developed to include negative numbers and the use of the four-quadrant number plane.
While alternative grid systems may be used in early experiences, it is intended that the standard rectangular grid system be established.

In this topic, linear refers to straight lines.
Investigate the use of coordinates by Descartes and Fermat to identify points in terms of positive or zero distances from axes. Isaac Newton introduced negative values.

## Language

Students will need to become familiar with and be able to use new terms including 'coefficient', 'constant term', and 'intercept'.

## Data

In our contemporary society, there is a constant need for all people to understand, interpret and analyse information displayed in tabular or graphical forms. Students need to recognise how information may be displayed in a misleading manner resulting in false conclusions.

The Data strand extends from Early Stage 1 to Stage 5.2 and includes the collection, organisation, display and analysis of data. Early experiences are based on real-life contexts using concrete materials. This leads to data collection methods and the display of data in a variety of ways. Students are encouraged to ask questions relevant to their experiences and interests and to design ways of investigating their questions. Students should be aware of the extensive use of statistics in society. Print and Internet materials are useful sources of data that can be analysed and evaluated. Tools such as spreadsheets and other software packages may be used where appropriate to organise, display and analyse data.

This section presents the outcomes, key ideas, knowledge and skills, and Working Mathematically statements from Early Stage 1 to Stage 3 in one substrand. The Stage 4 content is presented in the topics: Data Representation and Data Analysis and Evaluation.

## Early Stage 1

## Data

## DES1. 1

Represents and interprets data displays made from objects and pictures

## Key Ideas

Collect data about students and their environment
Organise actual objects or pictures of the objects into a data display
Interpret data displays made from objects and pictures

## Working Mathematically

## Students learn to

- pose questions about situations using everyday language eg 'What colour hair do most people in our class have?' (Questioning)
- interpret classroom data displays eg weather charts, behaviour charts (Reflecting, Communicating)
- give reasons why a column of three objects may look bigger than a column of five objects (Communicating, Reasoning)
- explain interpretations of information presented in data displays eg 'More children like dogs because there are more dog pictures than cat pictures.'
(Communicating, Reasoning)


## Background Information

At this Stage, students collect data about themselves and their environment with teacher assistance. Students use actual objects or pictures of the objects as data. They organise and present the data in groups or in rows.

The notion of representing an object with a different object is abstract and often difficult for students and is introduced in the next Stage.

Stage 1

## Data

## DS1.1

Gathers and organises data, displays data using column and picture graphs, and interprets the results

## Knowledge and Skills

## Students learn about

- gathering data and keeping track of what has been counted by using concrete materials, tally marks, words or symbols
- displaying data using concrete materials and pictorial representations
- using objects or pictures as symbols to represent data, using one-to-one correspondence eg using a block to represent each car
- using a baseline, equal spacing and same-sized symbols when representing data
- displaying data using column graphs and picture graphs
- interpreting information presented in picture graphs or column graphs


## Key Ideas

Gather and record data using tally marks
Display the data using concrete materials and pictorial representations
Use objects or pictures as symbols to represent other objects, using one-to-one correspondence
Interpret information presented in picture graphs and column graphs

## Working Mathematically

## Students learn to

- pose suitable questions that can be answered by gathering and displaying data eg "What will be the most popular colour of cars that pass the school in the next ten minutes?'( Questioning)
- determine what data to gather to investigate a question (Reasoning)
- predict the likely results of data to be collected (Reflecting)
- display data to communicate information gathered in other key learning areas eg data gathered in a unit on Mini Beasts (Communicating, Applying Strategies, Reflecting)
- use simple graphics software to create picture graphs (Applying Strategies)
- interpret data displayed in simple picture graphs and column graphs found in books and made by other students (Applying Strategies, Reflecting)
- identify misleading representations of data eg where the symbols are not the same size (Reflecting)


## Background Information

The notion of representing an object with a different object is abstract and is introduced at this Stage.
It is important that each object in a three-dimensional graph represents one object except in the case where things are used in pairs eg shoes. One object can also represent an idea such as one person's preference.

By collecting information to investigate a question, students can develop simple ways of recording. Some methods include

- placing blocks or counters in a line
- colouring squares on grid paper
- using tally marks.


## Language

Column graphs consist of vertical columns or horizontal bars. However, the term 'bar graph' is reserved for divided bar graphs and should not be used for a column graph with horizontal bars.

Stage 2

## Data

## DS2. 1

Gathers and organises data, displays data using tables and graphs, and interprets the results

## Key Ideas

Conduct surveys, classify and organise data using tables Construct vertical and horizontal column graphs and picture graphs
Interpret data presented in tables, column graphs and picture graphs

## Working Mathematically

## Students learn to

- pose a suitable question to be answered using a survey eg 'What is the most popular playground game among students in our class?' (Questioning)
- pose questions that can be answered using the information from a table or graph (Questioning)
- create a table to organise collected data, using a computer program eg spreadsheets (Applying Strategies)
- use simple graphing software to enter data and create a graph (Applying Strategies)
- interpret graphs found on the Internet, in media and in factual texts (Applying Strategies, Communicating)
- discuss the advantages and disadvantages of different representations of the same data (Communicating, Reflecting)
- compare tables and graphs constructed from the same data to determine which is the most appropriate method of display (Reasoning)


## Background Information

This topic provides many opportunities for students to collect information about a variety of areas of interest and can be readily linked with other key learning areas such as Human Society and Its Environment (HSIE) and Science.

Data could also be collected from the Internet.

## Language

Column graphs consist of vertical columns or horizontal bars. However, the term 'bar graph' is reserved for divided bar graphs and should not be used for a column graph with horizontal bars.

Stage 3

## Data

## DS3. 1

Displays and interprets data in graphs with scales of many-toone correspondence

## Key Ideas

Determine the mean (average) for a small set of data
Draw picture, column, line and divided bar graphs using scales of many-to-one correspondence
Read and interpret sector (pie) graphs
Read and interpret graphs with scales of many-to-one correspondence

## Working Mathematically

## Students learn to

- pose questions that can be answered using the information from a table or graph (Questioning)
- collect, represent and evaluate a set of data as part of an investigation, including data collected using the Internet (Applying Strategies)
- use a computer database to organise information collected from a survey (Applying Strategies)
- use a spreadsheet program to tabulate and graph collected data (Applying Strategies)
- determine what type of graph is the best one to display a set of data (Reflecting)
- explain information presented in the media that uses the term 'average' eg 'The average temperature for the month of December was 24 degrees.' (Communicating)
- discuss and interpret graphs found in the media and in factual texts (Communicating, Reflecting)
- identify misleading representations of data in the media (Reflecting)
- discuss the advantages and disadvantages of different representations of the same data (Communicating, Reflecting)


## Background Information

In picture graphs involving numbers that have a large range, one symbol cannot represent one real object.
A key is used for convenience eg ()$=10$ people.
Line graphs should only be used where meaning can be attached to the points on the line between plotted points.

Sector (pie) graphs and divided bar graphs are used to show how a total is divided into parts.
Column graphs are useful in recording the results obtained from simple probability experiments.
Advantages and disadvantages of different representations of the same data should be explicitly taught.

## Stage 4

## Data Representation

## DS4. 1

Constructs, reads and interprets graphs, tables, charts and statistical information

## Key Ideas

Draw, read and interpret graphs (line, sector, travel, step, conversion, divided bar, dot plots and stem-and-leaf plots), tables and charts
Distinguish between types of variables used in graphs
Identify misrepresentation of data in graphs
Construct frequency tables
Draw frequency histograms and polygons

## Working Mathematically

## Students learn to

- choose appropriate forms to display data (Communicating)
- write a story which matches a given travel graph (Communicating)
- read and comprehend a variety of data displays used in the media and in other school subject areas (Communicating)
- interpret back-to-back stem-and-leaf plots when comparing data sets (Communicating)
- analyse graphical displays to recognise features that may cause a misleading interpretation eg displaced zero, irregular scales (Communicating, Reasoning)
- compare the strengths and weaknesses of different forms of data display (Reasoning, Communicating)
- interpret data displayed in a spreadsheet (Communicating)
- identify when a line graph is appropriate (Communicating)
- interpret the findings displayed in a graph eg the graph shows that the heights of all children in the class are between 140 cm and 175 cm and that most are in the group 151-155 cm (Communicating)
- generate questions from information displayed in graphs (Questioning)


## Background Information

The construction of scales on axes can be linked with the drawing of similar figures in Space and Geometry.
It is important that students have the opportunity to gain experience with a wide range of tabulated and graphical data. Advantages and disadvantages of different representations of the same data should be explicitly taught.

Data may be quantitative (discrete or continuous) or categorical
eg gender (male, female) is categorical
height (measured in cm ) is quantitative, continuous quality (poor, average, good, excellent) is categorical school population (measured in individuals) is quantitative, discrete.

## Language

Students need to be provided with opportunities to discuss what information can be drawn from the data presented. Students need to think about the meaning of the information and to put it into their own words.

Language to be developed would include superlatives, comparatives and other language such as 'prefer .... over' etc.

## Stage 4

## Data Analysis and Evaluation

## DS4.2

Collects statistical data using either a census or a sample, and analyses data using measures of location and range

## Key Ideas

Use sampling and census
Make predictions from samples and diagrams
Analyse data using mean, mode, median and range

## Working Mathematically

## Students learn to

- work in a group to design and conduct an investigation eg - decide on an issue
- decide whether to use a census or sample
- choose appropriate methods of presenting questions (yes/no, tick a box, a scale of 1 to 5, open-ended, etc)
- analyse and present the data
- draw conclusions (Questioning, Reasoning, Applying Strategies, Communicating)
- use spreadsheets, databases, statistics packages, or other technology, to analyse collected data, present graphical displays, and discuss ethical issues that may arise from the data (Communicating, Applying Strategies)
- detect bias in the selection of a sample (Applying Strategies)
- consider the size of the sample when making predictions about the population (Applying Strategies)
- compare two sets of data by finding the mean, mode and/or median, and range of both sets (Applying Strategies)
- recognise that summary statistics may vary from sample to sample (Reasoning)
- draw conclusions based on the analysis of data (eg a survey of the school canteen food) using the mean, mode and/or median, and range (Applying Strategies, Reasoning)
- interpret media reports and advertising that quote various statistics eg media ratings (Communicating)
- question when it is more appropriate to use the mode or median, rather than the mean, when analysing data (Questioning)


## Background Information

Many school subjects make use of graphs and data eg in PDHPE students might review published statistics on road accidents, drownings etc.
In Stage 4 Design and Technology, students are required, in relation to marketing, to 'collect information about the needs of consumers in relation to each Design Project'.
The group investigation could relate to aspects of the PDHPE syllabus eg 'appraise the values and attitudes of society in relation to lifestyle and health'.
In Geography, range is used when discussing aspects such as temperature and is given by stating the maximum and minimum values. This is different to the use of 'range' in mathematics where the difference is calculated for the range.
In Geography, use is made of a computer database of local census data. Also, students collect information about global
climatic change, greenhouse gas emission, ozone depletion, acid rain, waste management and carbon emissions.
In Science, students carry out investigations to test or research a problem or hypothesis; they collect, record and analyse data and identify trends, patterns and relationships.
Many opportunities occur in this topic to implement aspects of the Key Competencies (see Cross-curriculum Content):

- collecting, analysing and organising information
- communicating ideas and information
- planning and organising activities
- working with others and in teams
- using mathematical ideas and techniques
- solving problems, and
- using technology.


## Measurement

Measurement enables the identification and quantification of attributes of objects so that they can be compared and ordered. All measurements are approximations; therefore opportunities to develop an understanding of approximation is important. Estimation skills are essential, particularly in situations where it is not convenient or necessary to use measuring devices. Accuracy in estimated measurement is obtained through extensive practice using a variety of units of measure and in a variety of contexts.

The Measurement strand for Early Stage 1 to Stage 3 is organised into five substrands that each focus on a particular attribute:

- Length
- Area
- Volume and Capacity
- Mass
- Time.

The development of each of these attributes progresses through several processes including identifying the attribute and making comparisons, using informal units, using formal units, and applying and generalising methods.

## Identifying the attribute and comparison

The first stage is recognising that objects have attributes that can be measured. Students begin by looking at, touching or directly comparing two or more objects in relation to a particular attribute. Through conversation and questioning students develop some of the language used to describe these attributes.

## Informal units

Students then continue to develop the key understandings of the measurement process using repeated informal units. Understandings include

- the need for repeated units that do not change
- the appropriateness of a selected unit
- the need for the same unit to be used to compare two or more objects
- the relationship between the size of the unit and the number required to measure, and
- the structure of the repeated units (for length, area and volume).


## Formal units

Discussions and comparisons of measurement with informal units will lead to the realisation that there is need for a standard unit. Experiences with formal units should allow students to:

- become familiar with the relative size of the unit
- determine the degree of accuracy required
- select and use the appropriate attribute and unit of measurement
- select and use the appropriate measuring device
- record and recognise the abbreviations, and
- convert between units.


## Applications and generalisations

Finally students apply this knowledge in a variety of contexts and begin to generalise their methods to calculate perimeters, areas and volumes.

This section presents the outcomes, key ideas, knowledge and skills, and Working Mathematically statements from Early Stage 1 to Stage 3 in each substrand. The Stage 4 content is presented in the topics Perimeter and Area, Surface Area and Volume, and Time.

## Early Stage 1

## Length

## MES1. 1

Describes length and distance using everyday language and compares lengths using direct comparison

## Key Ideas

Identify and describe the attribute of length
Compare lengths directly by placing objects side-by-side and aligning the ends
Record comparisons informally

## Working Mathematically

## Students learn to

- identify an object that is longer or shorter than another object eg 'Find an object longer than this pencil.' (Applying Strategies)
- predict whether an object will be longer or shorter than another object and explain their prediction (Reflecting, Reasoning)
- solve simple everyday problems using problem-solving strategies that include 'acting it out' (Applying Strategies)
- explain why the length of a piece of string remains unchanged if placed in a straight line or a curve (Communicating, Reasoning)
- use the attribute of length to make repeating patterns

(Applying Strategies, Reflecting)


## Background Information

At this Stage, students develop an awareness of what length is and some of the language used to describe length.
Students develop an awareness of the attribute of length as comparisons of lengths are made.
This Stage focuses on one-to-one comparisons and the importance of aligning the objects correctly at one end.
When students are asked to compare the lengths of two objects of equal length and can consistently say that the objects are equal in length though their relative positions have been altered, they are conserving length.


This is an important concept and develops over time. When students can compare two lengths they should then be given the opportunity to order three or more lengths. This process requires students to understand that if A is longer than $B$ and $B$ is longer than $C$, then $A$ is longer than $C$.
Distance and length are two distinct concepts. Activities should focus on concepts of length and distance.

## Language

Students may need to be given practice with the language of length in a variety of contexts. Students may know the word 'fat' but not the word 'thick'. Students may be using the general terms 'big' or 'long' for attributes such as height, width, depth, length and thickness.

Young students often confuse concepts such as big, tall, long and high. It is important to engage students in activities that help them differentiate between these concepts.

## Stage 1

## Length

## MS1.1

Estimates, measures, compares and records lengths and distances using informal units, metres and centimetres

## Key Ideas

Use informal units to estimate and measure length and distance by placing informal units end-to-end without gaps or overlaps
Record measurements by referring to the number and type of informal or formal units used
Recognise the need for metres and centimetres, and use them to estimate and measure length and distance

## Knowledge and Skills

## Students learn about

- using informal units to measure lengths or distances, placing the units end-to-end without gaps or overlaps
- counting informal units to measure lengths or distances, and describing the part left over
- comparing and ordering two or more lengths or distances using informal units
- estimating and measuring linear dimensions and curves using informal units
- recording lengths or distances by referring to the number and type of unit used
- describing why the length remains constant when units are rearranged
- making and using a tape measure calibrated in informal units eg calibrating a paper strip using footprints as a repeated unit
- recognising the need for a formal unit to measure lengths or distances
- using the metre as a unit to measure lengths or distances
- recording lengths and distances using the abbreviation for metre (m)
- measuring lengths or distances to the nearest metre or half-metre
- recognising the need for a smaller unit than the metre
- recognising that one hundred centimetres equal one metre
- using a 10 cm length, with 1 cm markings, as a device to measure lengths
- measuring lengths or distances to the nearest centimetre
- recording lengths and distances using the abbreviation for centimetre (cm)


## Working Mathematically

## Students learn to

- select and use appropriate informal units to measure lengths or distances eg using paper clips instead of popsticks to measure a pencil (Applying Strategies)
- explain the appropriateness of a selected informal unit (Communicating, Reflecting)
- use informal units to compare the lengths of two objects that cannot be moved or aligned (Applying Strategies)
- use computer software to draw a line and use a simple graphic as an informal unit to measure its length (Applying Strategies)
- explain the relationship between the size of a unit and the number of units needed eg more paper clips than popsticks will be needed to measure the length of the desk (Communicating, Reflecting)
- discuss strategies used to estimate length eg visualising the repeated unit (Communicating, Reflecting)
- explain that a metre length can be arranged in a variety of ways eg straight line, curved line (Communicating)


## Background Information

At this Stage, measuring the length of objects using informal units enables students to develop some key understandings of measurement. These include understanding:

- that units are repeatedly placed end-to-end without gaps or overlaps
- that units must be equal in size
- that identical units should be used to compare lengths
- that some units are more appropriate for measuring particular objects, and
- the relationship between the size of the unit and the number of units needed.
It is important that students have had some measurement experiences before being asked to estimate and that a variety of estimation strategies are taught.

Students should be given opportunities to apply their understandings of measurement, gained through experiences with informal units, to experiences with the centimetre and metre. Students could make a measuring device using informal units before using a ruler. This will assist students in understanding that the distances between marks on a ruler represent unit lengths and that the marks indicate the end points of each of the units.
At this Stage, making a measuring device from ten onecentimetre units and using it to measure allows students to count by tens and may be more manageable than a ruler.

Stage 2

## Length

## MS2. 1

Estimates, measures, compares and records lengths, distances and perimeters in metres, centimetres and millimetres

## Key Ideas

Estimate, measure, compare and record lengths and distances using metres, centimetres and/or millimetres
Estimate and measure the perimeter of two-dimensional shapes
Convert between metres and centimetres, and centimetres and millimetres
Record lengths and distances using decimal notation to two places

## Knowledge and Skills

## Students learn about

- describing one centimetre as one hundredth of a metre
- estimating, measuring and comparing lengths or distances using metres and centimetres
- recording lengths or distances using metres and centimetres eg 1 m 25 cm
- recognising the need for a smaller unit than the centimetre
- estimating, measuring and comparing lengths or distances using millimetres
- recognising that ten millimetres equal one centimetre and describing one millimetre as one tenth of a centimetre
- using the abbreviation for millimetre (mm)
- recording lengths or distances using centimetres and millimetres eg 5 cm 3 mm
- converting between metres and centimetres, and centimetres and millimetres
- recording lengths or distances using decimal notation to two decimal places eg 1.25 m
- recognising the features of an object associated with length, that can be measured eg length, breadth, height, perimeter
- using the term 'perimeter' to describe the total distance around a shape
- estimating and measuring the perimeter of twodimensional shapes
- using a tape measure, ruler or trundle wheel to measure lengths or distances


## Working Mathematically

## Students learn to

- describe how a length or distance was measured (Communicating)
- explain strategies used to estimate lengths or distances eg by referring to a known length (Communicating, Reflecting)
- select and use an appropriate device to measure lengths or distances (Applying Strategies)
- question and explain why two students may obtain different measures for the same length, distance or perimeter (Questioning, Communicating, Reasoning)
- explain the relationship between the size of a unit and the number of units needed eg more centimetres than metres will be needed to measure the same length (Communicating, Reflecting)


## Background Information

At this Stage, measurement experiences enable students to:

- develop an understanding of the size of the metre, centimetre and millimetre
- estimate and measure using these units, and
- select the appropriate unit and measuring device.


## Language

'Perimeter' comes from the Greek words that mean to measure around the outside.

Stage 3

## Length

## MS3. 1

Selects and uses the appropriate unit and device to measure lengths, distances and perimeters

## Key Ideas

Select and use the appropriate unit and device to measure lengths, distances and perimeters
Convert between metres and kilometres; millimetres, centimetres and metres
Record lengths and distances using decimal notation to three places
Calculate and compare perimeters of squares, rectangles and equilateral and isosceles triangles

## Working Mathematically

## Students learn to

- describe how a length or distance was estimated and measured (Communicating)
- explain the relationship between the size of a unit and the number of units needed eg more metres than kilometres will be needed to measure the same distance (Communicating, Reflecting)
- question and explain why two students may obtain different measures for the same length (Questioning, Communicating, Reasoning)
- interpret scales on maps and diagrams to calculate distances (Applying Strategies, Communicating)
- solve problems involving different units of length eg Find the total length of three items measuring 5 mm , 20 cm and 1.2 m . (Applying Strategies)
- explain that the perimeters of squares, rectangles and triangles can be found by finding the sum of the side lengths (Communicating, Reasoning)
- solve simple problems involving speed eg How long would it take to make a journey of 600 km if the average speed for the trip is $75 \mathrm{~km} / \mathrm{h}$ ?
(Applying Strategies)


## Background Information

When the students are able to measure efficiently and effectively using formal units, they should be encouraged to apply their knowledge and skills in a variety of contexts.

Following this they should be encouraged to generalise their method for calculating the perimeter of squares, rectangles and triangles.

## Language

'Perimeter' comes from the Greek words that mean to measure around the outside.

Early Stage 1

## Area

## MES1. 2

Describes area using everyday language and compares areas using direct comparison

## Knowledge and Skills

## Students learn about

- identifying the attribute of area as the measure of the amount of surface
- covering surfaces completely with smaller shapes
- making closed shapes and describing the area of the shape
- using everyday language to describe area eg surface, inside, outside
- using comparative language to describe area eg bigger than, smaller than, the same as
- estimating the larger of two areas and comparing by direct comparison eg superimposing
- recording area comparisons informally by drawing, tracing or cutting and pasting


## Key Ideas

Identify and describe the attribute of area
Estimate the larger of two areas and compare using direct comparison
Record comparisons informally

## Working Mathematically

## Students learn to

- ask questions about area in everyday situations eg 'Which book cover is bigger?'(Questioning)
- solve simple everyday problems using problem-solving strategies that include 'acting it out' (Applying Strategies)
- demonstrate how they determined which object has the biggest area (Communicating, Reasoning)
- explain why they think the area of one surface is bigger or smaller than another (Communicating, Reasoning)
- use computer software to draw a closed shape, colouring in the area (Applying Strategies)


## Background Information

At this Stage, students develop an awareness of what area is and some of the language used to describe area.
Area is the measure of the amount of surface. Surface refers to the outer faces or outside of an object. A surface may be flat or curved. Students develop an awareness of the attribute of area through covering activities, colouring in and as comparisons of area are made.
Students should be given opportunities to compare:

- two similar shapes of different size where one fits inside the boundary of the other
- two different-shaped objects where one can be placed on top of the other, and
- two shapes where one shape could be cut up and pasted onto the other.

When students can compare two areas they should then be given the opportunity to order three or more areas. This process requires students to understand that if A is larger than $B$ and $B$ is larger than $C$, then $A$ is larger than $C$.

Stage 1

## Area

## MS1. 2

Estimates, measures, compares and records areas using informal units

## Key Ideas

Use appropriate informal units to estimate and measure area
Compare and order two or more areas
Record measurements by referring to the number and type of informal units used

## Knowledge and Skills

## Students learn about

- comparing the areas of two surfaces that cannot be moved or superimposed eg by cutting paper to cover one surface and superimposing the paper over the second surface
- comparing the areas of two similar shapes by cutting and covering
- measuring area by placing identical informal units in rows or columns without gaps or overlaps
- counting informal units to measure area and describing the part left over
- estimating, comparing and ordering two or more areas using informal units
- drawing the spatial structure (grid) of the repeated units
- describing why the area remains constant when units are rearranged
- recording area by referring to the number and type of units used eg the area of this surface is 20 tiles


## Working Mathematically

## Students learn to

- select and use appropriate informal units to measure area (Applying Strategies)
- use computer software to create a shape and use a simple graphic as an informal unit to measure its area (Applying Strategies)
- explain why tessellating shapes are best for measuring area (Communicating, Reasoning)
- explain the structure of the unit tessellation in terms of rows and columns (Communicating)
- explain the relationship between the size of a unit and the number of units needed to measure area eg more tiles than workbooks will be needed to measure the area of the desktop (Communicating, Reflecting)
- discuss strategies used to estimate area eg visualising the repeated unit (Communicating, Reflecting)


## Background Information

Area is the measure of the amount of surface. Surface refers to the outer faces or outside of an object. A surface may be flat or curved.
At this Stage, measuring the area of objects using informal units enables students to develop some key understandings of measurement. These include repeatedly placing units so there are no gaps or overlaps and understanding that the units must be equal in size. Covering surfaces with a range of informal units should assist students in understanding that some units tessellate and are therefore more suitable for measuring area.

It is important that students have had some measurement experiences before being asked to estimate, and that a variety of estimation strategies is taught.
When students understand why tessellating units are important, they should be encouraged to make, draw and describe the spatial structure (grid).
Students should develop procedures for counting the tile or grid units so that no units are missed or counted twice. Students should also be encouraged to identify and use efficient strategies for counting eg using repeated addition or rhythmic counting.

Stage 2

## Area

## MS2. 2

Estimates, measures, compares and records the areas of surfaces in square centimetres and square metres

## Key Ideas

Recognise the need for square centimetres and square metres to measure area
Estimate, measure, compare and record areas in square centimetres and square metres

## Working Mathematically

## Students learn to

- question why two students may obtain different measurements for the same area (Questioning)


## Students learn about

- recognising the need for the square centimetre as a formal unit for measuring area
- using a $10 \mathrm{~cm} \times 10 \mathrm{~cm}$ tile (or grid) to find areas that are less than, greater than or about the same as 100 square centimetres
- estimating, measuring and comparing areas in square centimetres
- measuring a variety of surfaces using a square centimetre grid overlay
- recording area in square centimetres eg 55 square centimetres
- recognising the need for a unit larger than a square centimetre
- constructing a square metre
- estimating, measuring and comparing areas in square metres
- recording area in square metres eg 5 square metres
- using the abbreviations for square metre $\left(\mathrm{m}^{2}\right)$ and square centimetre $\left(\mathrm{cm}^{2}\right)$
- discuss and compare areas using some mathematical terms (Communicating)
- discuss strategies used to estimate area in square centimetres or square metres eg visualising repeated units (Communicating, Reflecting)
- apply strategies for measuring the areas of a variety of shapes (Applying Strategies)
- use efficient strategies for counting large numbers of square centimetres eg using strips of ten or squares of 100 (Applying Strategies)
- explain where square metres are used for measuring in everyday situations eg floor coverings
(Communicating, Reflecting)
- recognise areas that are 'smaller than', 'about the same as' and 'bigger than' a square metre (Applying Strategies)


## Background Information

At this Stage, students should appreciate that a formal unit allows for easier and more accurate communication of area measures.
Measurement experiences should enable students to develop an understanding of the size of units, select the appropriate unit, and estimate and measure using the unit.

An important understanding at this Stage is that an area of one square metre need not be a square. It could, for example, be a rectangle, two metres long and half a metre wide.

## Language

The abbreviation $\mathrm{m}^{2}$ is read 'square metre(s)' and not 'metre squared' or 'metre square'.

The abbreviation $\mathrm{cm}^{2}$ is read 'square centimetre(s)' and not 'centimetre squared' or 'centimetre square'.

Stage 3

## Area

## MS3.2

Selects and uses the appropriate unit to calculate area, including the area of squares, rectangles and triangles

## Knowledge and Skills

## Students learn about

- recognising the need for a unit larger than the square metre
- identifying situations where square kilometres are used for measuring area eg a suburb
- recognising and explaining the need for a more convenient unit than the square kilometre
- measuring an area in hectares eg the local park
- using the abbreviations for square kilometre $\left(\mathrm{km}^{2}\right)$ and hectare (ha)
- recognising that one hectare is equal to 10000 square metres
- selecting the appropriate unit to calculate area
- finding the relationship between the length, breadth and area of squares and rectangles
- finding the relationship between the base, perpendicular height and area of triangles
- reading and interpreting scales on maps and simple scale drawings to calculate an area
- finding the surface area of rectangular prisms by using a square centimetre grid overlay or by counting unit squares


## Key Ideas

Select and use the appropriate unit to calculate area Recognise the need for square kilometres and hectares Develop formulae in words for finding area of squares, rectangles and triangles

## Working Mathematically

## Students learn to

- apply measurement skills to everyday situations eg determining the area of the basketball court (Applying Strategies)
- use the terms 'length', 'breadth', 'width' and 'depth' appropriately (Communicating, Reflecting)
- extend mathematical tasks by asking questions eg 'If I change the dimensions of a rectangle but keep the perimeter the same, will the area change?' (Questioning)
- interpret measurements on simple plans (Communicating)
- investigate the areas of rectangles that have the same perimeter (Applying Strategies)
- explain that the area of rectangles can be found by multiplying the length by the breadth (Communicating, Reasoning)
- explain that the area of squares can be found by squaring the side length (Communicating, Reasoning)
- equate 1 hectare to the area of a square with side 100 m (Reflecting)


## Background Information

It is important at this Stage that students establish a real reference for the square kilometre and hectare eg locating a square kilometre or hectare area on a local map.
When the students are able to measure efficiently and effectively using formal units, they should be encouraged to apply their knowledge and skills in a variety of contexts.

Students could be encouraged to find more efficient ways of counting such as finding how many squares in one row and multiplying this by the number of rows.
Students should then begin to generalise their methods to calculate the area of rectangles and triangles. At this Stage, the formulae are described in words and not symbols.

Stage 4

## Perimeter and Area

## MS4. 1

Uses formulae and Pythagoras' theorem in calculating perimeter and area of circles and figures composed of rectangles and triangles

## Key Ideas

Describe the limits of accuracy of measuring instruments
Develop formulae and use to find the area and perimeter of triangles, rectangles and parallelograms
Find the areas of simple composite figures
Apply Pythagoras' theorem
Investigate and find the area and circumference of circles
Convert between metric units of length and area

## Knowledge and Skills

## Students learn about

## Length and Perimeter

- estimating lengths and distances using visualisation strategies
- recognising that all measurements are approximate
- describing the limits of accuracy of measuring instruments ( $\pm 0.5$ unit of measurement)
- interpreting the meaning of the prefixes 'milli', 'centi' and 'kilo'
- converting between metric units of length
- finding the perimeter of simple composite figures


## Pythagoras' Theorem

- identifying the hypotenuse as the longest side in any right-angled triangle and also as the side opposite the right angle
- establishing the relationship between the lengths of the sides of a right-angled triangle in practical ways, including the dissection of areas
- using Pythagoras' theorem to find the length of sides in right-angled triangles
- solving problems involving Pythagoras' theorem, giving an exact answer as a surd (eg $\sqrt{5}$ ) and approximating the answer using an approximation of the square root
- writing answers to a specified or sensible level of accuracy, using the 'approximately equals' sign
- identifying a Pythagorean triad as a set of three numbers such that the sum of the squares of the first two equals the square of the third
- using the converse of Pythagoras' theorem to establish whether a triangle has a right angle


## Areas of Squares, Rectangles, Triangles and Parallelograms

- developing and using formulae for the area of a square and rectangle
- developing (by forming a rectangle) and using the formula for the area of a triangle
- finding the areas of simple composite figures that may be dissected into rectangles and triangles


## Working Mathematically

## Students learn to

- consider the degree of accuracy needed when making measurements in practical situations (Applying Strategies)
- choose appropriate units of measurement based on the required degree of accuracy (Applying Strategies)
- make reasonable estimates for length and area and check by measuring (Applying Strategies)
- select and use appropriate devices to measure lengths and distances (Applying Strategies)
- discuss why measurements are never exact (Communicating, Reasoning)
- describe the relationship between the sides of a rightangled triangle (Communicating)
- use Pythagoras' theorem to solve practical problems involving right-angled triangles (Applying Strategies)
- apply Pythagoras' theorem to solve problems involving perimeter and area (Applying Strategies)
- identify the perpendicular height of triangles and parallelograms in different orientations (Communicating)
- find the dimensions of a square given its perimeter, and of a rectangle given its perimeter and one side length (Applying Strategies)
- solve problems relating to perimeter, area and circumference (Applying Strategies)
- compare rectangles with the same area and ask questions related to their perimeter such as whether they have the same perimeter
(Questioning, Applying Strategies, Reasoning)
- compare various shapes with the same perimeter and ask questions related to their area such as whether they have the same area
(Questioning, Applying Strategies, Reasoning)
- explain the relationship that multiplying, dividing, squaring and factoring have with the areas of squares and rectangles with integer side lengths (Reflecting)
- use mental strategies to estimate the circumference of circles, using an approximate value of $\pi$ eg 3
(Applying Strategies)


## Stage 4

## Perimeter and Area (continued)

- developing the formula by practical means for finding the area of a parallelogram eg by forming a rectangle using cutting and folding techniques
- converting between metric units of area
$1 \mathrm{~cm}^{2}=100 \mathrm{~mm}^{2}, 1 \mathrm{~m}^{2}=1000000 \mathrm{~mm}^{2}$, $1 \mathrm{ha}=10000 \mathrm{~m}^{2}, 1 \mathrm{~km}^{2}=1000000 \mathrm{~m}^{2}=100 \mathrm{ha}$


## Circumferences and Areas of Circles

- demonstrating by practical means that the ratio of the circumference to the diameter of a circle is constant eg by measuring and comparing the diameter and circumference of cylinders
- defining the number $\pi$ as the ratio of the circumference to the diameter of any circle
- developing, from the definition of $\pi$, formulae to calculate the circumference of circles in terms of the radius $r$ or diameter d
$C=\pi d$ or $C=2 \pi r$
- developing the formula by dissection and using it to calculate the area of circles $A=\pi r^{2}$
- find the area and perimeter of quadrants and semi-circles (Applying Strategies)
- find radii of circles given their circumference or area (Applying Strategies)
- solve problems involving $\pi$, giving an exact answer in terms of $\pi$ and an approximate answer using $\frac{22}{7}, 3.14$ or a calculator's approximation for $\pi$
(Applying Strategies)
- compare the perimeter of a regular hexagon inscribed in a circle with the circle's circumference to demonstrate that $\pi>3$ (Reasoning)


## Background Information

This topic links with substitution into formulae in Patterns and Algebra and rounding in Number.
Area and perimeter of quadrants and semicircles is linked with work on fractions.
Graphing of the relationship between a constant perimeter and possible areas of a rectangle is linked with Patterns and Algebra.
Finding the areas of rectangles and squares with integer side lengths is an important link between geometry and multiplication, division, factoring and squares. Factoring a number into the product of two numbers is equivalent to forming a rectangle with these side lengths, and squaring is equivalent to forming a square. Finding perimeters is in turn linked with addition and subtraction.
Students use measurement regularly in Science eg reading thermometers, using measuring cylinders, etc.
Students should develop a sense of the levels of accuracy that are appropriate to a particular situation eg the length of a bridge may be measured in metres to estimate a quantity of paint needed but would need to be measured far more accurately for engineering work.
Area formulae for the triangle and parallelogram need to be developed by practical means and related to the area of a rectangle. The rhombus is treated as a parallelogram and the area found using the formula $A=b h$.
Students should gain an understanding of Pythagoras' theorem, rather than just being able to recite the formula in words. By dissecting and rearranging the squares, they will appreciate that the theorem is a statement of a relationship amongst the areas of squares.
Pythagoras' theorem becomes, in Stage 5, the formula for the circle in the coordinate plane. These links can be developed later in the context of circle geometry and the trigonometry of the general angle.

The number $\pi$ is known to be irrational (not a fraction) and also transcendental (not the solution of any polynomial equation with integer coefficients). At this Stage, students only need to know that the digits in its decimal expansion do not repeat (all this means is that it is not a fraction), and in fact have no known pattern.
The formula for area of a circle may be established by using one or both of the following dissections:

- cut the circle into a large number of sectors, and arrange them alternately point-up and point-down to form a rectangle with height r and base length $\pi r$
- inscribe a number of congruent triangles in a circle, all with vertex at the centre and show that the area of the inscribed polygon is half the length of perimeter times the perpendicular height
- dissect the circle into a large number of concentric rings, cut the circle along a radius, and open it out to form a triangle with height $r$ and base $2 \pi r$.
Pythagoras' theorem was probably known many centuries before Pythagoras (c $580-\mathrm{c} 500 \mathrm{BC}$ ), to at least the Babylonians.
In the 1990s, Wiles finally proved a famous conjecture of Fermat (1601-1665), known as 'Fermat's last theorem', that says that if $n$ is an integer greater than 2 , then $a^{n}+b^{n}=c^{n}$ has no integer solution.
The Greek writer, Heron, is best known for his formula for the area of a triangle: $A=\sqrt{s(s-a)(s-b)(s-c)}$ where $\mathrm{a}, \mathrm{b}$ and c are the lengths of the sides of the triangle and $s$ is half the perimeter of the triangle.
$\operatorname{Pi}(\pi)$ is the Greek letter equivalent to ' p ', and is the first letter of the Greek word 'perimetron' meaning perimeter. In 1737, Euler used the symbol for pi for the ratio of the circumference to the diameter of a circle.
One of the three famous problems left unsolved by the ancient Greek mathematicians was the problem of 'squaring the circle' ie using straight edge and compasses to construct a square of area equal to a given circle.


## Early Stage 1

## Volume and Capacity

## MES1. 3

Compares the capacities of containers and the volumes of objects or substances using direct comparison

## Key Ideas

Identify and describe the attributes of volume and capacity Compare the capacities of two containers using direct comparison
Compare the volumes of two objects by direct observation Record comparisons informally

## Working Mathematically

## Students learn to

- recognise when a container is nearly full, half-full or empty (Applying Strategies)
Students learn about
- identifying the attribute of the volume of an object or substance as the amount of space it occupies
- identifying the attribute of the capacity of a container as the amount it can hold
- filling and emptying containers using materials such as water, sand, marbles and blocks
- using the terms 'full', 'empty' and 'about half-full'
- using comparative language to describe volume and capacity eg has more, has less, will hold more, will hold less
- stacking and packing blocks into defined spaces eg boxes, cylindrical cans
- comparing the capacities of two containers directly by
- filling one and pouring into the other
- packing materials from one container into the other
- comparing the volumes of two piles of material by filling two identical containers
- comparing the volumes of two objects by directly observing the amount of space each occupies eg a garbage truck takes up more space than a car
- using drawings, numerals and words to record volume and capacity comparisons informally
- recognise and explain which three-dimensional objects pack and stack easily (Communicating, Reflecting)
- question and predict whether an object or collection of objects will fit inside a defined space such as a box or cupboard (Ouestioning, Applying Strategies, Reflecting)
- solve simple everyday problems using problem-solving strategies that include 'acting it out' (Applying Strategies)
- predict which container has the greater capacity (Applying Strategies)


## Background Information

Volume and capacity relate to the measurement of threedimensional space, in the same way that area relates to the measurement of two-dimensional space.
Volume refers to the amount of space occupied by an object or substance.
Capacity refers to the amount a container can hold. Capacity is only used in relation to containers.
At this Stage, comparisons are made directly using methods such as pouring or packing the contents of one container into another.

Early experiences often lead students to the conclusion that taller containers 'hold more'. To develop beyond this, students need to directly compare containers that are:

- short and hold more; tall and hold less
- short and hold less; tall and hold more
- short and hold the same as a tall container.

Many opportunities to emphasise volume and capacity concepts occur when students pack toys or objects into cupboards, or in play situations eg sand pit, water play.

## Language

The term 'big' is often used by students to describe a variety of attributes. Depending on the context, it could mean long, tall, heavy, etc.

It is important to model more precise language with students to describe volume or capacity.

## Stage 1

## Volume and Capacity

## MS1.3

Estimates, measures, compares and records volumes and capacities using informal units

## Key Ideas

Use appropriate informal units to estimate and measure volume and capacity
Compare and order the capacities of two or more containers and the volumes of two or more models or objects
Record measurements by referring to the number and type of informal units used

## Working Mathematically

## Students learn to

- explain a strategy used for estimating capacity or volume (Communicating)
- select an appropriate informal unit to measure and compare the capacities of two containers eg using cups rather than teaspoons to fill a bucket (Applying Strategies)
- explain that if a smaller unit is used then more units are needed to measure eg more cups than ice cream containers are needed to fill a bucket (Communicating, Reasoning)
- solve simple everyday problems using problem-solving strategies including trial and error (Applying Strategies)
- devise and explain strategies for packing and counting units to fill a box eg packing in layers and ensuring there are no gaps between units
(Communicating, Applying Strategies)
- recognise that cubes pack and stack better than other shapes (Reflecting)
- recognise that containers of different shapes may have the same capacity (Reflecting)
- recognise that models with different appearances may have the same volume (Reflecting)
- recognise that changing the shape of an object does not change the amount of water it displaces (Reflecting)
- packing cubic units (eg blocks) into rectangular containers so there are no gaps
- estimating the volume of a pile of material and checking by measuring
- comparing and ordering the volumes of two or more models by counting the number of blocks used in each model
- comparing and ordering the volumes of two or more objects by marking the change in water level when each is submerged
- recording volume or capacity by referring to the number and type of informal units used


## Background Information

Volume refers to the amount of space occupied by an object or substance. Capacity refers to the amount a container can hold. Capacity is only used in relation to containers.
Students need experience in filling containers with both continuous material (eg water) and with discrete objects (eg marbles or blocks).
The use of continuous material leads to measurement using the litre and millilitre in later Stages.

The use of blocks leads to measurement using the units of cubic metre and cubic centimetre.
Calibrating a container using informal units is a precursor to students using measuring cylinders calibrated in formal units (litres and millilitres) at a later Stage.
An object displaces its own volume when totally submerged. Links with fractions using $\frac{1}{2}$ and $\frac{1}{4}$ cups to fill containers.

## Language

The word 'volume' has different meanings in everyday contexts eg volume in relation to sound levels, a volume of a book.

Students need meaningful practice in using the general word 'container' to include bottles, jars, tubs, etc.

Stage 2

## Volume and Capacity

MS2.3 - Unit 1 (litres and cubic centimetres)
Estimates, measures, compares and records volumes and capacities using litres, millilitres and cubic centimetres

## Key Ideas

Recognise the need for a formal unit to measure volume and capacity
Estimate, measure, compare and record volumes and capacities using litres
Measure the volume of models in cubic centimetres

## Working Mathematically

## Students learn to

- explain the need for a standard unit to measure the volume of liquids and the capacity of containers (Communicating)
- estimate the number of cups needed to fill a container with a capacity of one litre (Applying Strategies)
- recognise that one litre containers can be a variety of shapes (Reflecting)
- relate the litre to familiar everyday containers eg milk cartons (Reflecting)
- interpret information about capacity and volume on commercial packaging (Communicating, Reflecting)
- estimate the volume of a substance in a partially filled container from the information on the label detailing the contents of the container (Applying Strategies)
- distinguish between mass and volume eg 'This stone is heavier than the ball but it takes up less room.' (Reflecting)


## Background Information

At this Stage, students should appreciate that a formal unit allows for easier and more accurate communication of measures and are introduced to the litre, cubic centimetre and millilitre.
Measurement experiences should enable students to develop an understanding of the size of the unit, estimate and measure using the unit, and select the appropriate unit and measuring device.

Fluids are commonly measured in litres and millilitres. Hence the capacities of containers used to hold fluids are usually measured in litres and millilitres eg a litre of milk will fill a container whose capacity is 1 litre.
The cubic centimetre can be introduced and related to the centimetre as a unit to measure length and the square centimetre as a unit to measure area.

## Language

The abbreviation $\mathrm{cm}^{3}$ is read 'cubic centimetre(s)' and not 'centimetre cubed'.

## Stage 2

## Volume and Capacity

## MS2.3 - Unit 2 (millilitres and displacement)

Estimates, measures, compares and records volumes and capacities using litres, millilitres and cubic centimetres

## Key Ideas

Estimate, measure, compare and record volumes and capacities using litres and millilitres
Convert between litres and millilitres

## Working Mathematically

## Students learn to

- explain the need for a standard unit to measure the volume of liquids and the capacity of containers (Communicating)
- estimate and measure quantities to the nearest 100 mL and/or to the nearest 10 mL (Applying Strategies)
- interpret information about capacity and volume on commercial packaging (Communicating, Reflecting)
- estimate the volume of a substance in a partially filled container from the information on the label detailing the contents of the container (Applying Strategies)
- relate the millilitre to familiar everyday containers and familiar informal units eg 1 teaspoon is approximately $5 \mathrm{~mL}, 250 \mathrm{~mL}$ fruit juice containers (Reflecting)
- estimate the change in water level expected when an object is submerged (Applying Strategies)


## Background Information

The displacement strategy for finding the volume of an object relies on the fact that an object displaces its own volume when it is totally submerged in a liquid.

The strategy may be applied in two ways:

- using a partially filled, calibrated, clear container and noting the change in the level of the liquid when the object is submerged, or
- submerging an object into a container filled to the brim with liquid and measuring the overflow.


## Language

The abbreviation $\mathrm{cm}^{3}$ is read 'cubic centimetre(s)' and not 'centimetres cubed'.

Stage 3

## Volume and Capacity

## MS3. 3

Selects and uses the appropriate unit to estimate and measure volume and capacity, including the volume of rectangular prisms

## Knowledge and Skills

## Students learn about

- constructing rectangular prisms using cubic centimetre blocks and counting to determine volume
- estimating then measuring the capacity of rectangular containers by packing with cubic centimetre blocks
- recognising the need for a unit larger than the cubic centimetre
- using the cubic metre as a formal unit for measuring larger volumes
- using the abbreviation for cubic metre $\left(\mathrm{m}^{3}\right)$
- estimating the size of a cubic metre, half a cubic metre and two cubic metres
- selecting the appropriate unit to measure volume and capacity
- using repeated addition to find the volume of rectangular prisms
- finding the relationship between the length, breadth, height and volume of rectangular prisms
- calculating the volume of rectangular prisms
- demonstrating that a cube of side 10 cm will displace 1 L of water
- demonstrating, by using a medicine cup, that a cube of side 1 cm will displace 1 mL of water
- equating 1 cubic centimetre to 1 millilitre and 1000 cubic centimetres to 1 litre
- finding the volume of irregular solids in cubic centimetres using a displacement strategy
- recording volume and capacity using decimal notation to three decimal places eg 1.275 L


## Background Information

Volume refers to the space occupied by an object or substance. Capacity refers to the amount a container can hold. Capacity is only used in relation to containers.
It is not necessary to refer to these definitions with students.
When the students are able to measure efficiently and effectively using formal units, they could use centimetre cubes to construct rectangular prisms, counting the number of cubes to determine

## Key Ideas

Recognise the need for cubic metres
Estimate and measure the volume of rectangular prisms
Select the appropriate unit to measure volume and capacity
Determine the relationship between cubic centimetres and millilitres
Record volume and capacity using decimal notation to three decimal places

## Working Mathematically

## Students learn to

- explain the advantages of using a cube as a unit to measure volume (Communicating, Reasoning)
- explain that objects with the same volume may have different shapes (Communicating, Reflecting)
- construct different rectangular prisms that have the same volume (Applying Strategies)
- recognise that an object that displaces 300 mL of water has a volume of 300 cubic centimetres (Reflecting)
- explain why volume is measured in cubic metres in certain situations eg wood bark, concrete (Communicating, Reasoning)
- estimate the number of cubic metres in a variety of objects such as a cupboard, a car, a bus, the classroom (Applying Strategies)
- explain that the volume of rectangular prisms can be found by finding the number of cubes in one layer and multiplying by the number of layers
(Applying Strategies, Reflecting)


## Language

The abbreviation $\mathrm{m}^{3}$ is read 'cubic metre(s)' and not 'metres cubed'.
volume and then begin to generalise their method for calculating the volume.
The cubic metre can be introduced and related to the metre as a unit to measure length and the square metre as a unit to measure area. It is important that students are given opportunities to reflect on their understanding of length and area so they can use this to calculate volume.

## Stage 4

## Surface Area and Volume

## MS4.2

Calculates surface area of rectangular and triangular prisms and volume of right prisms and cylinders

## Key Ideas

Find the surface area of rectangular and triangular prisms
Find the volume of right prisms and cylinders
Convert between metric units of volume

## Working Mathematically

## Students learn to

- solve problems involving surface area of rectangular and triangular prisms (Applying Strategies)
- solve problems involving volume and capacity of right prisms and cylinders (Applying Strategies)
- recognise, giving examples, that prisms with the same volume may have different surface areas, and prisms with the same surface area may have different volumes (Reasoning, Applying Strategies)


## Background Information

This outcome is linked with the properties of solids treated in the Space and Geometry strand. It is important that students can visualise rectangular and triangular prisms in different orientations before they find the surface area or volume. They should be able to sketch other views of the object.
The volumes of rectangular prisms and cubes are linked with multiplication, division, factorisation and powers. Factoring a number into the product of three numbers is equivalent to forming a rectangular prism with these side lengths, and to forming a cube if the numbers are all equal. Some students may be interested in knowing what fourth and higher powers, and the
product of four or more numbers, correspond to.
When developing the volume formula students require an understanding of the idea of cross-section and can visualise, for example, stacking unit cubes layer by layer into a rectangular prism, or stacking planks into a pile.
The focus here is on right prisms and cylinders, although the formulae for volume also apply to oblique prisms and cylinders provided the perpendicular height is used. Refer to the Background Information in SGS4.1 Properties of Solids (page 122 ) for definitions of right and oblique prisms and cylinders.

## Early Stage 1

## Mass

## MES1.4

Compares the masses of two objects and describes mass using everyday language

## Knowledge and Skills

## Students learn about

- identifying the attribute of mass as the amount of matter in an object
- describing objects in terms of their mass eg heavy, light, hard to push, hard to pull
- using comparative language to describe mass eg heavier, lighter, heaviest, lightest
- comparing and describing two masses by pushing or pulling
- comparing two masses directly by hefting eg 'This toy feels heavier than that one.'
- sorting objects on the basis of their mass
- using an equal arm balance to compare the masses of two objects
- identifying materials that are light or heavy
- using drawings and words to record mass comparisons informally


## Key Ideas

Identify and describe the attribute of mass
Compare the masses of two objects by pushing, pulling or hefting or using an equal arm balance
Record comparisons informally

## Working Mathematically

## Students learn to

- ask questions about why they can or cannot lift an object (Questioning)
- predict which object would be heavier than, lighter than or have about the same mass as another object (Applying Strategies)
- give reasons why they think one object will be heavier than another (Reasoning)
- check a prediction about the masses of two objects by using an equal arm balance (Applying Strategies)
- discuss the action of an equal arm balance when a heavy object is placed in one pan and a lighter object in the other pan (Communicating)


## Background Information

At this Stage, students develop an awareness of the attribute of mass and some of the language used to describe mass.
Opportunities to explore mass concepts and understand the action of an equal arm balance occur in play situations.
'Hefting' is the balancing of objects, holding one in each hand and deciding which is the heavier or lighter. At this Stage students should be comparing only two objects that are quite different in mass.

Early experiences often lead students to the conclusion that large things are heavier than small things and if two things are the same size and shape then they will have the same mass. To develop beyond this, students need to have experiences with objects that are:

- light and large : heavy and large
- light and small : heavy and small
- large but lighter than a smaller object.

When students realise that changing the shape of an object does not alter its mass they are said to conserve the property of mass.

## Language

As the terms 'weigh' and 'weight' are common in everyday usage, they can be accepted in student language should they arise. Weight is a force which changes with gravity, while mass remains constant.

Stage 1

## Mass

## MS1. 4

Estimates, measures, compares and records the masses of two or more objects using informal units

## Key Ideas

Estimate and measure the mass of an object using an equal arm balance and appropriate informal units
Compare and order two or more objects according to mass Record measurements by referring to the number and type of informal units used

## Knowledge and Skills

## Students learn about

- comparing and ordering the masses of two or more objects by hefting and then checking using an equal arm balance
- placing two objects on either side of an equal arm balance to obtain a level balance
- measuring the mass of an object by counting the number of informal units needed to balance the object
- estimating and recording mass by referring to the number and type of informal units used
- comparing and ordering the masses of two or more objects using informal units
- using an equal arm balance to find two collections of objects that have the same mass eg a collection of blocks and a collection of counters
- calculating differences in mass by measuring and comparing eg 'The pencil has a mass equal to three blocks and a pair of plastic scissors has a mass of six blocks, so the scissors are three blocks heavier than the pencil.'


## Working Mathematically

## Students learn to

- predict whether the measure will be greater or smaller when a different unit is used (Applying Strategies)
- select an appropriate informal unit to measure the mass of an object and justify the choice (Applying Strategies)
- solve a variety of problems using problem-solving strategies, including trial and error (Applying Strategies)
- explain why some informal units are more appropriate in a given situation (Communicating, Reasoning)
- ask questions related to the size and mass of objects eg 'Why is this small wooden block heavier than this empty plastic bottle?' (Questioning)
- recognise that mass is conserved eg the mass of a lump of plasticine remains constant regardless of shape (Reflecting)


## Background Information

Mass is an intrinsic property of an object, but its most common measure is in terms of weight. Weight is a force that changes with gravity, while mass remains constant.
At this Stage, measuring mass using informal units enables students to develop some key understandings of measurement. These include:

- repeatedly using a unit as a measuring device
- selecting an appropriate unit for a specific task
- appreciating that a common informal unit is necessary for comparing the mass of objects, and
- understanding that some units are unsatisfactory because they are not uniform eg pebbles.

Students should appreciate that the equal arm balance has two functions

- comparing the mass of two objects
- measuring the mass of an object by repeatedly using a unit as a measuring device.
When comparing and measuring collections of objects, students may focus on quantity rather than mass eg students may comment that five ping-pong balls are heavier than one small metal ball.
It is important that students have had some measurement experiences before being asked to estimate and that a variety of estimation strategies are taught.


## Language

As the terms 'weigh' and 'weight' are common in everyday usage, they can be accepted in student language should they arise.

Stage 2

## Mass

## MS2.4

Estimates, measures, compares and records masses using kilograms and grams

## Key Ideas

Recognise the need for a formal unit to measure mass Estimate, measure, compare and record masses using kilograms and grams

## Knowledge and Skills

## Students learn about

- recognising the need for a formal unit to measure mass
- using the kilogram as a unit to measure mass
- using hefting to identify objects that are 'more than', 'less than' and 'about the same as' one kilogram
- measuring the mass of an object in kilograms using an equal arm balance
- estimating and checking the number of similar objects that have a total mass of one kilogram
- using the abbreviation for kilogram (kg)
- recognising the need for a unit smaller than the kilogram
- measuring and comparing the masses of objects in kilograms and grams using a set of scales
- using the abbreviation for grams (g)
- recognising that 1000 grams equal one kilogram
- interpreting commonly used fractions of a kilogram including $\frac{1}{2}, \frac{1}{4}, \frac{3}{4}$ and relating these to the number of grams


## Working Mathematically

## Students learn to

- recognise that objects with a mass of one kilogram can be a variety of shapes and sizes (Reflecting)
- interpret statements, and discuss the use of grams and kilograms, on commercial packaging (Communicating)
- discuss strategies used to estimate mass eg by referring to a known mass (Communicating)
- question and explain why two students may obtain different measures for the same mass (Questioning, Communicating, Reasoning)
- solve problems including those involving commonly used fractions of a kilogram (Applying Strategies)


## Background Information

At this Stage, students should appreciate that a formal unit allows for easier and more accurate communication of mass measures and are introduced to the kilogram and gram.

Students should develop an understanding of the size of these units, and estimate and measure using the units.

Stage 3

## Mass

## MS3.4

Selects and uses the appropriate unit and measuring device to find the mass of objects

## Knowledge and Skills

## Students learn about

- recognising the need for a unit larger than the kilogram
- using the tonne to record large masses eg sand, soil, vehicles
- using the abbreviation for tonne ( t )
- converting between kilograms and grams and between kilograms and tonnes
- selecting and using the appropriate unit and device to measure mass
- recording mass using decimal notation to three decimal places eg 1.325 kg
- relating the mass of one litre of water to one kilogram


## Key Ideas

Recognise the need for tonnes
Convert between kilograms and grams and between kilograms and tonnes
Select and use the appropriate unit and device to measure mass

Record mass using decimal notation to three decimal places

## Working Mathematically

## Students learn to

- solve problems involving different units of mass eg Find the total mass of three items weighing 50 g , 750 g and 2.5 kg . (Applying Strategies)
- associate gram measures with familiar objects eg a standard egg has a mass of about 60 g (Communicating)
- find the approximate mass of a small object by establishing the mass of a number of that object eg 'The stated weight of a box of chocolates is 250 g . If there are 20 chocolates in the box, what does each chocolate weigh?' (Applying Strategies)


## Background Information

Gross mass is the mass of the contents and the container. Nett mass is the mass of the contents only.

Local industry could provide a source for the study of measurement in tonnes eg weighbridges, cranes and hoists.

## Language

'Mass' and' weight' have become interchangeable in everyday usage.

## Early Stage 1

## Time

## MES1.5

Sequences events and uses everyday language to describe the duration of activities

## Knowledge and Skills

## Students learn about

- using and understanding terms such as 'daytime', 'nighttime', 'yesterday', 'today', 'tomorrow', 'before', 'after', 'next', 'morning' and 'afternoon'
- sequencing events in time
- comparing the duration of two events using informal methods eg 'It takes me longer to eat my lunch than it does to clean my teeth.'
- recalling that there are seven days in a week
- naming and ordering the days of the week
- relating events to a particular day or time of day eg 'Assembly is on Tuesday', 'We come to school in the morning.'
- naming the seasons
- classifying week-days and weekend days
- reading hour time on a digital and an analog clock
- using the term 'o'clock'


## Key Ideas

Describe the duration of events using everyday language
Sequence events in time
Name days of the week and seasons
Tell time on the hour on digital and analog clocks

## Working Mathematically

## Students learn to

- ask questions related to time eg 'How long is it until lunchtime?', 'Is tomorrow Wednesday?' (Questioning)
- describe events that take 'a long time' and events that take 'a short time' (Communicating, Reflecting)
- identify events that occur every day eg 'We have news every day.' (Reflecting)
- describe the position of the hands on an analog clock when reading hour time (Communicating)


## Background Information

The focus on hour time at this Stage is only a guide. Some students will be able to read other times.

## Duration

At this Stage, students begin to develop an understanding of the duration of time as well as identify moments in time.
An understanding of duration is introduced through ideas such as 'before', 'after', 'how long' and 'how soon'. It should be noted that time spans at this Stage are personal judgements. Moments in time include ideas such as 'day-time', 'today', days of the week and seasons.

Sunday is the first day of the calendar week. A week, however, may begin on any day eg 'The week beginning the fourth of May.'
Teachers should be aware of the multicultural nature of our society and of the significant times in the year for different cultural groups. These could include religious festival days, national days, sporting events and anniversaries.

## Telling Time

At this Stage, 'telling time' focuses on reading the hour on both analog and digital clocks.

## Language

The words 'long' and 'short' can be confusing to students who have only experienced these words in terms of length measurement. Students will need experience with these words in both length and time contexts.

References to time are often incorrectly used in everyday language eg 'I'll be a second', 'back in a minute'.

## Stage 1

## Time

## MS1.5

Compares the duration of events using informal methods and reads clocks on the half-hour

## Key Ideas

Use informal units to measure and compare the duration of events
Name and order the months and seasons of the year
Identify the day and date on a calendar
Tell time on the hour and half-hour on digital and analog clocks

## Working Mathematically

## Students learn to

- discuss activities that take one hour, less than an hour, more than an hour (Communicating, Reflecting)
- indicate when it is thought that an activity has gone for one hour, one minute or one second (Applying Strategies)
- solve simple everyday problems using problem-solving strategies, including:
- trial and error
- drawing a diagram
(Applying Strategies, Communicating)
- describe the position of the hands on a clock for the halfhour (Communicating)
- associate everyday events with particular hour and halfhour times eg 'We start school at 9 o'clock.'
(Reflecting)


## Background Information

'Timing' and 'telling time' are two different notions. The first relates to the duration of time and the second is 'dial reading'. Both, however, assist students in understanding the passage of time and its measurement.

## Duration

At this Stage, the focus is on the passage of time measured using informal units and in hours, minutes and seconds. Using informal units allows students to focus on the process of repeatedly using a unit as a measuring device.
It is important at this Stage to have students develop a sense of one hour, one minute and one second through practical experiences rather than know that there are 60 minutes in an hour.

## Telling Time

At this Stage, 'telling time' focuses on reading the half-hour on both analog and digital clocks. An important understanding is that when the minute hand shows the half-hour, the hour hand is always half-way between two hour markers.
Students need to be aware that there are three ways of expressing the time.


Note: When writing digital time, two dots should separate hours and minutes eg 9:30.

## Language

The terms 'hour hand' and 'minute hand' should be used rather than 'big hand' and 'little hand' to promote understanding of their respective functions.

Stage 2

## Time

## MS2.5

Reads and records time in one-minute intervals and makes comparisons between time units

## Key Ideas

Recognise the coordinated movements of the hands on a clock
Read and record time using digital and analog notation
Convert between units of time
Read and interpret simple timetables, timelines and calendars

## Knowledge and Skills

## Students learn about

- recognising the coordinated movements of the hands on an analog clock, including
- how many minutes it takes for the minute hand to move from one numeral to the next
- how many minutes it takes for the minute hand to complete one revolution
- how many minutes it takes for the hour hand to move from one numeral to the next
- how many minutes it takes for the minute hand to move from the twelve to any other numeral
- how many seconds it takes for the second hand to complete one revolution
- associating the numerals 3,6 and 9 with 15,30 and 45 minutes and using the terms 'quarter-past' and 'quarter-to'
- identifying which hour has just passed when the hour hand is not pointing to a numeral
- reading analog and digital clocks to the minute eg 7:35 is read as 'seven thirty-five'
- recording digital time using the correct notation eg 9:15
- relating analog notation to digital notation eg ten to nine is the same as $8: 50$
- converting between units of time
eg 60 seconds $=1$ minute
60 minutes $=1$ hour
24 hours = 1 day
- reading and interpreting simple timetables, timelines and calendars


## Working Mathematically

## Students learn to

- recall time facts eg 24 hours in a day (Communicating, Applying Strategies)
- discuss time using appropriate language (Communicating)
- solve a variety of problems using problem-solving strategies, including:
- trial and error
- drawing a diagram
- working backwards
- looking for patterns
- using a table
(Applying Strategies, Communicating)
- record in words various times as shown on analog and digital clocks (Communicating)
- compare and discuss the relationship between time units eg an hour is a longer time than a minute (Communicating, Reflecting)


## Background Information

Discuss with students the use of informal units of time and their use in other cultures, including the use of Aboriginal time units.

A solar year actually lasts 365 days 5 hours 48 minutes and 45.7 seconds.

Stage 3

## Time

## MS3.5

Uses twenty-four hour time and am and pm notation in reallife situations and constructs timelines

## Key Ideas

Convert between am/pm notation and 24-hour time
Compare various time zones in Australia, including during daylight saving
Draw and interpret a timeline using a scale
Use timetables involving 24-hour time

## Knowledge and Skills

## Students learn about

- using am and pm notation
- telling the time accurately using 24 -hour time eg ' 2330 is the same as $11: 30 \mathrm{pm}$ '
- converting between 24 -hour time and am or pm notation
- determining the duration of events using starting and finishing times to calculate elapsed time
- using a stopwatch to measure and compare the duration of events
- comparing various time zones in Australia, including during daylight saving
- reading, interpreting and using timetables from real-life situations, including those involving 24 -hour time
- determining a suitable scale and drawing a timeline using the scale
- interpreting a given timeline using the scale


## Working Mathematically

## Students learn to

- explain where 24 -hour time is used eg transport, armed forces, VCRs (Communicating, Reflecting)
- select the appropriate unit to measure time and order a series of events according to the time taken to complete them (Applying Strategies)
- determine the local times in various time zones in Australia (Applying Strategies)
- use bus, train, ferry, and airline timetables, including those accessed on the Internet, to prepare simple travel itineraries (Applying Strategies)
- use a number of strategies to solve unfamiliar problems, including:
- trial and error
- drawing a diagram
- working backwards
- looking for patterns
- simplifying the problem
- using a table
(Applying Strategies, Communicating)


## Background Information

Australia is divided into three time zones. Time in Queensland, New South Wales, Victoria and Tasmania is Eastern Standard Time (EST), time in South Australia and the Northern Territory is half an hour behind EST, and time in Western Australia is two hours behind EST.
The terms 'am' and 'pm' are used only for the digital form of time recording and not with the 'o'clock' terminology.
The abbreviation am stands for the Latin words 'ante meridiem' which means 'before midday'. The abbreviation $p m$ stands for 'post meridiem' which means 'after midday'.

Midday and midnight need not be expressed in am or pm form. ' 12 noon' or ' 12 midday' and ' 12 midnight' should be used, even though 12:00 pm and 12:00 am are sometimes seen.
It is important to note that there are many different ways of recording dates, including abbreviated forms. Different notations for dates are used in different countries eg $8{ }^{\text {th }}$ December 2002 is recorded as 8.12 .02 in Australia but as 12.8.02 in America.

Stage 4

## Time

## MS4. 3

Performs calculations of time that involve mixed units

## Knowledge and Skills

## Students learn about

- adding and subtracting time mentally using bridging strategies eg from 2:45 to 3:00 is 15 minutes and from 3:00 to 5:00 is 2 hours, so the time from 2:45 until 5:00 is 15 minutes +2 hours $=2$ hours 15 minutes
- adding and subtracting time with a calculator using the 'degrees, minutes, seconds' button
- rounding calculator answers to the nearest minute or hour
- interpreting calculator displays for time calculations eg 2.25 on a calculator display for time means $2 \frac{1}{4}$ hours
- comparing times and calculating time differences between major cities of the world eg "Given that London is 10 hours behind Sydney, what time is it in London when it is 6:00 pm in Sydney?'
- interpreting and using tables relating to time eg tide charts, sunrise/sunset tables, bus, train and airline timetables, standard time zones


## Key Ideas

Perform operations involving time units
Use international time zones to compare times
Interpret a variety of tables and charts related to time

## Working Mathematically

## Students learn to

- plan the most efficient journey to a given destination involving a number of connections and modes of transport (Applying Strategies)
- ask questions about international time relating to everyday life eg whether a particular soccer game can be watched live on television during normal waking hours ( Questioning)
- solve problems involving calculations with mixed time units eg 'How old is a person today if he/she was born on 30/6/1989?' (Applying Strategies)


## Background Information

Time has links with work on rates involving time eg speed. The calculation of time can be done on a scientific calculator and links with fractions and decimals.

This topic could be linked to the timing of track and swimming events in the PDHPE syllabus.

The Babylonians thought that the Earth took 360 days to travel around the Sun (last centuries BC). This is why there are $360^{\circ}$ in one revolution and hence $90^{\circ}$ in one right angle. There are 60 minutes ( $60^{\prime}$ ) in one hour and 60 minutes in one degree. The word 'minute' (meaning 'small') and minute (time measure), although pronounced differently, are really the same word. A minute (time) is a minute (small) part of one hour. A minute (angle) is a minute (small) part of a right angle.

## Space and Geometry

Space and Geometry is the study of spatial forms. It involves representation of shape, size, pattern, position and movement of objects in the three-dimensional world, or in the mind of the learner.

The Space and Geometry strand for Early Stage 1 to Stage 3 is organised into three substrands:

- Three-dimensional space
- Two-dimensional space
- Position

The Space and Geometry strand enables the investigation of three-dimensional objects and two-dimensional shapes as well as the concepts of position, location and movement. Important and critical skills for students to acquire are those of recognising, visualising and drawing shapes and describing the features and properties of three-dimensional objects and two-dimensional shapes in static and dynamic situations. Features are generally observable whereas properties require mathematical knowledge eg 'a rectangle has four sides' is a feature and 'a rectangle has opposite sides of equal length' is a property. Manipulation of a variety of real objects and shapes is crucial to the development of appropriate levels of imagery, language and representation.

When classifying quadrilaterals, teachers need to be aware of the inclusivity of the classification system. That is, trapeziums are inclusive of the parallelograms, which are inclusive of the rectangles and rhombuses, which are inclusive of the squares. These relationships are presented in the following Venn diagram, which is included here as background information.


For example, a rectangle is a special type of parallelogram. It is a parallelogram that contains a right angle. A rectangle may also be considered to be a trapezium that has both pairs of opposite sides parallel and equal.

This section presents the outcomes, key ideas, knowledge and skills, and Working Mathematically statements from Early Stage 1 to Stage 3 in each substrand. The Stage 4 content is presented in the topics of Properties of Solids, Angles, and Properties of Geometrical Figures.

## Early Stage 1

## Three-dimensional Space

## SGES1.1

Manipulates, sorts and represents three-dimensional objects and describes them using everyday language

## Key Ideas

Manipulate and sort three-dimensional objects found in the environment
Describe features of three-dimensional objects using everyday language
Use informal names for three-dimensional objects

## Working Mathematically

## Students learn to

- manipulate and describe a hidden object using everyday language eg describe an object hidden in a 'mystery bag' (Applying Strategies, Communicating)
- use everyday language to describe the sorting of objects (Communicating)
- recognise and explain how a group of objects has been sorted eg 'These objects are all pointy.' (Applying Strategies, Reasoning, Communicating)
- predict the building and stacking capabilities of threedimensional objects (Applying Strategies)
- use a plank or board to find out which objects roll and which objects slide (Applying Strategies)
- describe the difference between three-dimensional objects and two-dimensional shapes using everyday language (Communicating, Reflecting)


## Background Information

At this Stage, the emphasis is on students handling, describing, sorting and representing the many objects around them. It is important that students are encouraged to use their own language to discuss and describe these objects.

Manipulation of a variety of real objects and shapes is crucial to the development of appropriate levels of imagery, language and representation.

## Language

Teachers can model mathematical language while still accepting and encouraging students' informal terms.

## Stage 1

## Three-dimensional Space

## SGS1. 1

Sorts, describes and represents three-dimensional objects including cones, cubes, cylinders, spheres and prisms, and recognises them in pictures and the environment

## Key Ideas

Name, describe, sort and model cones, cubes, cylinders, spheres and prisms
Recognise three-dimensional objects in pictures and the environment, and presented in different orientations
Recognise that three-dimensional objects look different from different views

## Knowledge and Skills

## Students learn about

- manipulating and describing common three-dimensional objects including cones, cubes, cylinders, spheres and prisms
- identifying and naming three-dimensional objects including cones, cubes, cylinders, spheres and prisms from a collection of everyday objects
- identifying cones, cubes, cylinders, spheres and prisms presented in different orientations

- recognising three-dimensional objects from pictures, photographs and in the environment
- using the terms 'faces', 'edges' and 'corners' to describe three-dimensional objects
- identifying two-dimensional shapes as faces of threedimensional objects
- sorting three-dimensional objects according to particular attributes eg shape of faces
- representing three-dimensional objects by making simple models, drawing or painting
- recognising that three-dimensional objects look different from different views eg a cup, a cone


## Working Mathematically

## Students learn to

- explain the attribute or multiple attributes used when sorting three-dimensional objects (Reasoning)
- select an object from a description of its features eg find an object with six square faces (Applying Strategies, Communicating)
- represent three-dimensional objects using a variety of materials, including computer drawing tools (Applying Strategies)
- use materials, pictures, imagery and actions to describe the features of three-dimensional objects (Applying Strategies, Communicating)
- explain or demonstrate how a simple model was made (Reasoning, Communicating)


## Background Information

At this Stage, students begin to explore objects in greater detail. They continue to describe the objects using their own language and are introduced to some formal language. Developing and retaining mental images of objects is an important skill for these students.

Manipulation of a variety of real objects and shapes in the classroom, the playground and outside the school is crucial to the development of appropriate levels of imagery, language and representation.

## Language

The mathematical term for a corner of a three-dimensional object is 'vertex'. The plural is 'vertices'. At this Stage, students may use the everyday term 'corner'.

The word 'face' has different meanings in different contexts. In mathematics the term 'face' refers to a flat surface eg a cube has six faces.

Stage 2

## Three-dimensional Space

## SGS2.1

Makes, compares, describes and names three-dimensional objects including pyramids, and represents them in drawings

## Knowledge and Skills

## Students learn about

- comparing and describing features of prisms, pyramids, cylinders, cones and spheres
- identifying and naming three-dimensional objects as prisms, pyramids, cylinders, cones and spheres
- recognising similarities and differences between prisms, pyramids, cylinders, cones and spheres
- identifying three-dimensional objects in the environment and from drawings, photographs or descriptions
- making models of prisms, pyramids, cylinders, cones and spheres given a three-dimensional object, picture or photograph to view
- sketching prisms, pyramids, cylinders and cones, attempting to show depth
- creating nets from everyday packages eg a cereal box
- sketching three-dimensional objects from different views including top, front and side views
- making and visualising the resulting cut face (plane section) when a three-dimensional object receives a straight cut
- recognising that prisms have a uniform cross-section when the section is parallel to the base
- recognising that pyramids do not have a uniform crosssection


## Background Information

The formal names for particular prisms and pyramids are not introduced at this Stage. Prisms and pyramids are to be treated as classes to group all prisms and all pyramids. Names for particular prisms or pyramids are introduced in Stage 3.
Prisms have two bases that are the same shape and size. The bases of a prism may be squares, rectangles, triangles or other polygons. The other faces in the net are rectangular if the faces are perpendicular to the base. The base of a prism is the shape of the uniform cross-section, not necessarily the face on which it is resting.
Pyramids differ from prisms in that they have only one base and all the other faces are triangular. The triangular faces meet at a common vertex.
A section is a representation of an object as it would appear if cut by a plane eg if the corner was cut off a cube, the resulting cut face would be a triangle. An important understanding at

## Key Ideas

Name, describe, sort, make and sketch prisms, pyramids, cylinders, cones and spheres
Create nets from everyday packages
Describe cross-sections of three-dimensional objects

## Working Mathematically

## Students learn to

- describe three-dimensional objects using everyday language and mathematical terminology (Communicating)
- recognise and describe the use of three-dimensional objects in a variety of contexts eg buildings, packaging (Reflecting, Communicating)
- compare features of three-dimensional objects and twodimensional shapes
(Applying Strategies, Reflecting)
- compare own drawings of three-dimensional objects with other drawings and photographs of three-dimensional objects (Reflecting)
- explore, make and describe the variety of nets that can be used to create particular three-dimensional objects (Applying Strategies, Reasoning, Communicating)
- draw three-dimensional objects using a computer drawing package, attempting to show depth
(Applying Strategies)
this Stage is that the cross-sections parallel to the base of prisms are uniform and the cross-sections parallel to the base of pyramids are not.
Students could explore these ideas by stacking uniform objects to model prisms, and stacking sets of seriated shapes to model pyramids. (Note: such stacks are not strictly pyramids but assist understanding)
eg


In Geometry a three-dimensional object is called a solid. The three-dimensional object may in fact be hollow but it is still defined as a geometrical solid.
Models at this Stage should include skeletal models.

## Stage 3

## Three-dimensional Space

## SGS3. 1

Identifies three-dimensional objects, including particular prisms and pyramids, on the basis of their properties, and visualises, sketches and constructs them given drawings of different views

## Key Ideas

Identify three-dimensional objects, including particular prisms and pyramids, on the basis of their properties
Construct three-dimensional models given drawings of different views

## Knowledge and Skills

## Students learn about

- recognising similarities and differences between pyramids or prisms eg between a triangular prism and a hexagonal prism
- naming prisms or pyramids according to the shape of their base eg rectangular prism, hexagonal prism
- identifying and listing the properties of three-dimensional objects
- visualising and sketching three-dimensional objects from different views
- constructing three-dimensional models given drawings of different views
- visualising and sketching nets for three-dimensional objects
- showing simple perspective in drawings by showing depth


## Working Mathematically

## Students learn to

- explain why particular three-dimensional objects are used in the built environment or appear in the natural environment (Communicating, Reflecting)
- describe to a peer how to construct or draw a threedimensional object (Communicating)
- reflect on own drawing of a three-dimensional object and consider whether it can be improved (Reflecting)
- ask questions about shape properties when identifying them (Questioning)


## Background Information

At this Stage, the formal names for particular prisms and pyramids (eg rectangular prism, hexagonal pyramid) are introduced while students are engaged in their construction and representation. Only 'family' names were introduced in the previous Stage eg prism.
It is important that geometrical terms are not over-emphasised at the expense of understanding the concepts that the terms represent.

Students at this Stage are continuing to develop their skills of visual imagery, including the ability to:

- perceive and hold an appropriate mental image of an object or arrangement, and
- predict the shape of an object that has been moved or altered.

Stage 4

## Properties of Solids

## SGS4. 1

Describes and sketches three-dimensional solids including polyhedra, and classifies them in terms of their properties

## Knowledge and Skills

## Students learn about

- describing solids in terms of their geometric properties
number of faces
shape of faces
number and type of congruent faces
number of vertices
number of edges
convex or non-convex
- identifying any pairs of parallel flat faces of a solid
- determining if two straight edges of a solid are intersecting, parallel or skew
- determining if a solid has a uniform cross-section
- classifying solids on the basis of their properties

A polyhedron is a solid whose faces are all flat.
A prism has a uniform polygonal cross-section.
A cylinder has a uniform circular cross-section.
A pyramid has a polygonal base and one further vertex (the apex).

A cone has a circular base and an apex.
All points on the surface of a sphere are a fixed distance from its centre.

- identifying right prisms and cylinders and oblique prisms and cylinders
- identifying right pyramids and cones and oblique pyramids and cones
- sketching on isometric grid paper shapes built with cubes
- representing three-dimensional objects in two dimensions from different views
- confirming, for various convex polyhedra, Euler's formula

$$
F+V=E+2
$$

relating the number of faces $(F)$, the number of vertices $(V)$ and the number of edges ( $E$ )

- exploring the history of Platonic solids and how to make models of them
- making models of polyhedra


## Key Ideas

Determine properties of three-dimensional objects
Investigate Platonic solids
Investigate Euler's relationship for convex polyhedra
Make isometric drawings

## Working Mathematically

## Students learn to

- interpret and make models from isometric drawings (Communicating)
- recognise solids with uniform and non-uniform crosssections (Communicating)
- analyse three-dimensional structures in the environment to explain why they may be particular shapes eg buildings, packaging (Reasoning)
- visualise and name a common solid given its net (Communicating)
- recognise whether a diagram is a net of a solid (Communicating)
- identify parallel, perpendicular and skew lines in the environment (Communicating, Reflecting)


## Stage 4

## Properties of Solids (continued)

## Background Information

The volumes, surface areas and edge lengths of solids are a continuing topic of the Measurement strand.
The description above of the cone is not a strict definition unless one adds here, 'and every interval from the apex to a point on the circular edge lies on the curved surface'. For most students it would be inappropriate to raise the issue.
In a right prism, the base and top are perpendicular to the other faces. In a right pyramid or cone, the base has a centre of rotation, and the interval joining that centre to the apex is perpendicular to the base (and thus is its axis of rotation).
Oblique prisms, cylinders, pyramids and cones are those that are not right.
A polyhedron is called regular if its faces are congruent regular polygons and all pairs of adjacent faces make equal angles with each other. There are only five regular polyhedra: the regular tetrahedron, hexahedron (cube), octahedron, dodecahedron and icosahedron. They are also known as the Platonic solids, because Plato used them in his description of the nature of matter. Each can be drawn in a sphere, and a sphere can be drawn inside each.

Polyhedra have three types of boundaries - faces, edges and vertices. Euler's formula gives a relationship amongst the numbers of these boundaries for convex polyhedra. The formula does not always hold if the solid has curved faces or is non-convex.
Students could investigate when and where Plato and Euler lived and their contributions to mathematics.
Students in Years 7-10 Design and Technology may apply the skills developed in this topic, when they 'prepare diagrams, sketches and/or drawings for the making of models or products'.
The Years 7-10 Design and Technology Syllabus, in the section on graphical communication, refers to sketching, drawing with instruments, technical drawing, isometric drawing, orthographic drawing and perspective drawing.
In Science students investigate shapes of crystals. This may involve drawing and building models of crystals.
This topic may be linked to perspective drawing in art work.

## Early Stage 1

## Two-dimensional Space

## SGES1.2

Manipulates, sorts and describes representations of twodimensional shapes using everyday language

## Key Ideas

Manipulate, sort and describe two-dimensional shapes Identify and name circles, squares, triangles and rectangles in pictures and the environment, and presented in different orientations
Represent two-dimensional shapes using a variety of materials Identify and draw straight and curved lines

## Knowledge and Skills

## Students learn about

- identifying and drawing straight and curved lines
- comparing and describing closed shapes and open lines
- manipulating circles, squares, triangles and rectangles, and describing features using everyday language
- sorting two-dimensional shapes according to features, including size and shape
- identifying, representing and naming circles, squares, triangles and rectangles presented in different orientations

- identifying circles, squares, triangles and rectangles in pictures and the environment
- making representations of two-dimensional shapes using a variety of materials, including paint, paper, body movements and computer drawing tools
- drawing a two-dimensional shape by tracing around one face of a three-dimensional object


## Background Information

Experiences with shapes, even from this Stage, should not be limited. It is important that students experience shapes that are

- represented in a variety of ways eg 'tall skinny’ triangles, 'short fat' triangles, right-angled triangles
- presented in different orientations
- different sizes, and
- represented using a variety of materials eg paint, images on the computer, string.

Manipulation of a variety of real objects and shapes is crucial to the development of appropriate levels of imagery, language and representation.
Students should be given time to explore materials and represent shapes by tearing, painting, drawing, writing, or cutting and pasting.

## Stage 1

## Two-dimensional Space

## SGS1. 2

Manipulates, sorts, represents, describes and explores various two-dimensional shapes

## Key Ideas

Identify, name, compare and represent hexagons, rhombuses and trapeziums presented in different orientations
Make tessellating designs using flips, slides and turns
Identify a line of symmetry
Identify and name parallel, vertical and horizontal lines
Identify corners as angles
Compare angles by placing one angle on top of another

## Working Mathematically

## Students learn to

- select a shape from a description of its features (Applying Strategies, Communicating)
- visualise, make and describe recently seen shapes (Applying Strategies, Communicating)
- describe objects in their environment that can be represented by two-dimensional shapes (Communicating, Reflecting)
- identify shapes that are embedded in an arrangement of shapes or in a design (Applying Strategies)
- explain the attribute used when sorting two-dimensional shapes (Communicating)
- use computer drawing tools to complete a design with one line of symmetry (Applying Strategies)
- create a picture or design using computer paint, draw and graphics tools (Applying Strategies)
- manipulate an image using computer functions including 'flip', 'move', 'rotate' and 'resize' (Applying Strategies)
- describe the movement of a shape as a single flip, slide or turn (Communicating)
- recognise that the name of a shape doesn't change by changing its orientation in space (Reflecting)
- making tessellating designs by flipping, sliding and turning a two-dimensional shape
- identifying shapes that do, and do not, tessellate
- identifying and naming parallel, vertical and horizontal lines in pictures and the environment
- identifying the arms and vertex of the angle in a corner
- comparing angles by placing one angle on top of another


## Background Information

Manipulation of a variety of real objects and shapes is crucial to the development of appropriate levels of imagery, language and representation. Skills of visualising three-dimensional objects and two-dimensional shapes are developing at this Stage and must be fostered through practical activities and communication.

It is important for students to experience a broad range and variety of shapes in order to develop flexible mental images.

Students need to be able to recognise shapes presented in different orientations. In addition, they should have experiences identifying both regular and irregular shapes. Regular shapes have all sides equal.
A shape is said to have symmetry if both parts match when it is folded along a line of symmetry. Each part is the mirror image of the other.

## Language

The term 'arm' has different meanings in different contexts.

Stage 2

## Two-dimensional Space

## SGS2.2a

Manipulates, compares, sketches and names two-dimensional shapes and describes their features

## Key Ideas

Identify and name pentagons, octagons and parallelograms presented in different orientations
Compare and describe special groups of quadrilaterals
Make tessellating designs by reflecting, translating and rotating
Find all lines of symmetry for a two-dimensional shape

## Working Mathematically

## Students learn to

- select a shape from a description of its features (Applying Strategies, Communicating)
- describe objects in the environment that can be represented by two-dimensional shapes (Communicating, Reflecting)
- explain why a particular two-dimensional shape has a given name eg 'It has four sides, and the opposite sides are parallel.' (Communicating, Reflecting)
- recognise that a particular shape can be represented in different sizes and orientations (Reflecting)
- use computer drawing tools to create a tessellating design by copying, pasting and rotating regular shapes (Applying Strategies)
- describe designs in terms of reflecting, translating and rotating (Communicating)
- explain why any line through the centre of a circle is a line of symmetry (Communicating, Reasoning)
- determine that a triangle cannot be constructed from three straws if the sum of the lengths of the two shortest straws is less than the longest straw (Reasoning)
- explain how four straws of different lengths can produce quadrilaterals of different shapes and also threedimensional figures (Communicating, Reasoning)
- explain why a four-sided frame is not rigid (Communicating, Reasoning)
- making tessellating designs by reflecting (flipping), translating (sliding) and rotating (turning) a twodimensional shape
- finding lines of symmetry for a given shape


## Background Information

It is important for students to experience a variety of shapes in order to develop flexible mental images.
Students need to be able to recognise shapes presented in different orientations. In addition, they should have experiences identifying both regular and irregular shapes. Regular shapes have all sides equal and all angles equal.

When constructing polygons using materials such as straws of different lengths for sides, students should be guided to an understanding that:

- sometimes a triangle cannot be made from 3 straws
- a shape made from three lengths, ie a triangle, is always flat
- a shape made from four or more lengths need not be flat
- a unique triangle is formed if given three lengths
- more than one two-dimensional shape will result if more than three lengths are used.


## Language

It is actually the angles that are the focus for the general naming system used for shapes. A polygon (Greek 'many angles') is a closed shape with three or more angles and sides.

Quadrilateral is a term used to describe all four-sided figures.

Stage 2

## Two-dimensional Space

## SGS2.2b

Identifies, compares and describes angles in practical situations

## Knowledge and Skills

## Students learn about

- identifying and naming perpendicular lines
- identifying angles with two arms in practical situations eg corners
- identifying the arms and vertex of the angle in an opening, a slope and a turn where one arm is visible eg the bottom of a door when it is open is the visible arm and the imaginary line at the base of the doorway is the other arm
- comparing angles using informal means such as an angle tester
- describing angles using everyday language and the term 'right' to describe the angle formed when perpendicular lines meet
- drawing angles of various sizes by tracing along the adjacent sides of shapes and describing the angle drawn


## Key Ideas

Recognise openings, slopes and turns as angles
Describe angles using everyday language and the term 'right'
Compare angles using informal means

## Working Mathematically

## Students learn to

- identify examples of angles in the environment and as corners of two-dimensional shapes (Applying Strategies, Reflecting)
- identify angles in two-dimensional shapes and threedimensional objects (Applying Strategies)
- create simple shapes using computer software involving direction and angles (Applying Strategies)
- explain why a given angle is, or is not, a right angle (Reasoning)


## Background Information

At this Stage, students need informal experiences of creating, identifying and describing a range of angles. This will lead to an appreciation of the need for a formal unit to measure angles which is introduced in Stage 3.
The use of informal terms 'sharp' and 'blunt' to describe acute and obtuse angles respectively are actually counterproductive in identifying the nature of angles as they focus students' attention to the external points of the angle rather than the amount of turning between the angle arms.
Paper folding is a quick and simple means of generating a wide range of angles for comparison and copying.

A simple angle tester can be created by cutting the radii of two equal circles and sliding the cuts together. Another can be made by joining two narrow straight pieces of card with a split-pin to form the rotatable arms of an angle.


The arms of these angles are different lengths. However, the angles are the same size as the amount of turning between the arms is the same.
Students may mistakenly judge an angle to be greater in size than another on the basis of the length of the arms of the angles in the diagram.

## Language

Polygons are named according to the number of angles eg pentagons have five angles, hexagons have six angles, and octagons have eight angles.

Stage 3

## Two-dimensional Space

## SGS3.2a

Manipulates, classifies and draws two-dimensional shapes and describes side and angle properties

## Key Ideas

Identify right-angled, isosceles, equilateral and scalene triangles Identify and draw regular and irregular two-dimensional shapes
Identify and name parts of a circle
Enlarge and reduce shapes, pictures and maps
Identify shapes that have rotational symmetry

## Working Mathematically

## Students learn to

- select a shape from a description of its features (Applying Strategies, Communicating)
- describe side and angle properties of two-dimensional shapes (Communicating)
- construct a shape using computer drawing tools, from a description of its side and angle properties (Applying Strategies)
- explain classifications of two-dimensional shapes (Communicating)
- inscribe squares, equilateral triangles, regular hexagons and regular octagons in circles (Applying Strategies)
- explain the difference between regular and irregular shapes (Communicating)
- construct designs with rotational symmetry, including using computer drawing tools (Applying Strategies)
- enlarge or reduce a graphic or photograph using computer software (Applying Strategies)
- use computer drawing tools to manipulate shapes in order to investigate rotational symmetry
(Applying Strategies)


## Background Information

A shape is said to have rotational symmetry if a tracing of the shape matches it after the tracing is rotated part of a full turn.

## Language

Scalene means ‘uneven’ (Greek word ‘skalenos’: uneven): our English word 'scale' comes from the same word. Isosceles comes from the two Greek words 'isos': equals and 'skelos': leg;
equilateral comes from the two Latin words 'aequus': equal and 'latus': side; equiangular comes from 'aequus' and another Latin word 'angulus': corner.

Stage 3

## Two-dimensional Space

## SGS3.2b

Measures, constructs and classifies angles

## Key Ideas

Classify angles as right, acute, obtuse, reflex, straight or a revolution
Measure in degrees and construct angles using a protractor

## Knowledge and Skills

## Students learn about

- identifying the arms and vertex of an angle where both arms are invisible, such as rotations and rebounds
- recognising the need for a formal unit for the measurement of angles
- using the symbol for degrees $\left({ }^{\circ}\right)$
- using a protractor to construct an angle of a given size and to measure angles
- estimating and measuring angles in degrees
- classifying angles as right, acute, obtuse, reflex, straight or a revolution
- identifying angle types at intersecting lines


## Working Mathematically

## Students learn to

- describe angles found in their environment (Communicating, Reflecting)
- compare angles in different two-dimensional shapes (Applying Strategies)
- explain how an angle was measured (Communicating)
- rotate a graphic or object through a specified angle about a particular point, including using the rotate function in a computer drawing program
(Applying Strategies)


## Background Information

A circular protractor calibrated from $0^{\circ}$ to $360^{\circ}$ may be easier for students to use to measure reflex angles than a semicircular protractor calibrated from $0^{\circ}$ to $180^{\circ}$. There are $360^{\circ}$ in an angle of complete revolution.

A rebound could be created by rolling a tennis ball towards a wall at an angle and tracing the path with chalk to show the angle.

## Stage 4

## Angles

## SGS4.2

Identifies and names angles formed by the intersection of straight lines, including those related to transversals on sets of parallel lines, and makes use of the relationships between them

## Key Ideas

Classify angles and determine angle relationships Construct parallel and perpendicular lines and determine associated angle properties
Complete simple numerical exercises based on geometrical properties

## Working Mathematically

## Students learn to

- recognise and explain why adjacent angles adding to $90^{\circ}$ form a right angle (Reasoning)
- recognise and explain why adjacent angles adding to $180^{\circ}$ form a straight angle (Reasoning)
- recognise and explain why adjacent angles adding to $360^{\circ}$ form a complete revolution (Reasoning)
- find the unknown angle in a diagram using angle results, giving reasons
(Applying Strategies, Reasoning)
- apply angle results to construct a pair of parallel lines using a ruler and a protractor, a ruler and a set square, or a ruler and a pair of compasses
(Applying Strategies)
- apply angle and parallel line results to determine properties of two-dimensional shapes such as the square, rectangle, parallelogram, rhombus and trapezium (Applying Strategies, Reasoning, Reflecting)
- identify parallel and perpendicular lines in the environment (Reasoning, Reflecting)
- construct a pair of perpendicular lines using a ruler and a protractor, a ruler and a set square, or a ruler and a pair of compasses (Applying Strategies)
- use dynamic geometry software to investigate angle relationships
(Applying Strategies, Reasoning)


## Background Information

At this Stage, students are to be encouraged to give reasons when finding unknown angles. For some students formal setting out could be introduced. For example,
$\angle \mathrm{ABO}=70^{\circ}$ (corresponding angles, $\mathrm{AC} \| \mathrm{PR}$ )
Eratosthenes' calculation of the circumference of the earth used parallel line results.

Students could explore the results about angles associated with parallel lines cut by a transversal by starting with
corresponding angles - move one vertex and all four angles to the other vertex by a translation. The other two results then follow using vertically opposite angles and angles on a straight line. Alternatively, the equality of the alternate angles can be seen by rotation about the midpoint of the transversal.

## Stage 4

## Properties of Geometrical Figures

## SGS4. 3

Classifies, constructs, and determines the properties of triangles and quadrilaterals

## Key Ideas

Classify, construct and determine properties of triangles and quadrilaterals
Complete simple numerical exercises based on geometrical properties

## Working Mathematically

## Students learn to

- sketch and label triangles and quadrilaterals from a given verbal description (Communicating)
- describe a sketch in sufficient detail for it to be drawn (Communicating)
- recognise that a given triangle may belong to more than one class (Reasoning)
- recognise that the longest side of a triangle is always opposite the largest angle
(Applying Strategies, Reasoning)
- recognise and explain why two sides of a triangle must together be longer than the third side (Applying Strategies, Reasoning)
- recognise special types of triangles and quadrilaterals embedded in composite figures or drawn in various orientations (Communicating)
- determine if particular triangles and quadrilaterals have line and/or rotational symmetry (Applying Strategies)
- apply geometrical facts, properties and relationships to solve numerical problems such as finding unknown sides and angles in diagrams (Applying Strategies)
- justify their solutions to problems by giving reasons using their own words (Reasoning)
- bisect an angle by applying geometrical properties eg constructing a rhombus (Applying Strategies)
- bisect an interval by applying geometrical properties eg constructing a rhombus (Applying Strategies)
- draw a perpendicular to a line from a point on the line by applying geometrical properties eg constructing an isosceles triangle (Applying Strategies)
- draw a perpendicular to a line from a point off the line by applying geometrical properties eg constructing a rhombus (Applying Strategies)
- use ruler and compasses to construct angles of $60^{\circ}$ and $120^{\circ}$ by applying geometrical properties eg constructing an equilateral triangle (Applying Strategies)
- explain that a circle consists of all points that are a given distance from the centre and how this relates to the use of a pair of compasses
(Communicating, Reasoning)
- use dynamic geometry software to investigate the properties of geometrical figures (Applying Strategies, Reasoning)


## Quadrilaterals

- distinguishing between convex and non-convex quadrilaterals (the diagonals of a convex quadrilateral lie inside the figure)
- establishing that the angle sum of a quadrilateral is $360^{\circ}$
- constructing various types of quadrilaterals
- investigating the properties of special quadrilaterals (trapeziums, kites, parallelograms, rectangles, squares and rhombuses) by using symmetry, paper folding, measurement and/or applying geometrical reasoning. Properties to be considered include :
opposite sides parallel
opposite sides equal
adjacent sides perpendicular
opposite angles equal
diagonals equal in length
diagonals bisect each other
diagonals bisect each other at right angles diagonals bisect the angles of the quadrilateral


## Stage 4

## Properties of Geometrical Figures (continued)

- investigating the line symmetries and the order of rotational symmetry of the special quadrilaterals
- classifying special quadrilaterals on the basis of their properties


## Circles

- identifying and naming parts of the circle and related lines, including arc, tangent and chord
- investigating the line symmetries and the rotational symmetry of circles and of diagrams involving circles, such as a sector and a circle with a chord or tangent


## Background Information

The properties of special quadrilaterals are important in Measurement. For example, the perpendicularity of the diagonals of a rhombus and a kite allow a rectangle of twice the size to be constructed around them, leading to formulae for finding area.
At this Stage, the treatment of triangles and quadrilaterals is still informal, with students consolidating their understandings of different triangles and quadrilaterals and being able to identify them from their properties.
Students who recognise class inclusivity and minimum requirements for definitions may address this Stage 4 outcome concurrently with Stage 5 Space and Geometry outcomes, where properties of triangles and quadrilaterals are deduced from formal definitions.

Students should be encouraged to give reasons orally and in written form for their findings and answers. For some students formal setting out could be introduced.
A range of examples of the various triangles and quadrilaterals should be given, including quadrilaterals containing a reflex angle and figures presented in different orientations.
Mathematical templates and software such as dynamic geometry, and draw and paint packages are additional tools that are useful in drawing and investigating geometrical figures. Computer drawing programs enable students to prepare tessellation designs and to compare these with other designs such as those of Escher.

## Language

Scalene means 'uneven' (Greek word 'skalenos': uneven): our English word 'scale' comes from the same word. Isosceles comes from the two Greek words 'isos': equals and 'skelos': leg; equilateral comes from the two Latin words 'aequus': equal and 'latus': side; equiangular comes from 'aequus' and another Latin word 'angulus': corner.

## Stage 4

## Properties of Geometrical Figures

## SGS4. 4

Identifies congruent and similar two-dimensional figures stating the relevant conditions

## Key Ideas

Identify congruent figures
Investigate similar figures and interpret and construct scale drawings

## Working Mathematically

## Students learn to

- recognise congruent figures in tessellations, art and design work (Reflecting)
- interpret and use scales in photographs, plans and drawings found in the media and/or other learning areas (Applying Strategies, Communicating)
- enlarge diagrams such as cartoons and pictures (Applying Strategies)
- apply similarity to finding lengths in the environment where it is impractical to measure directly eg heights of trees, buildings (Applying Strategies, Reasoning)
- apply geometrical facts, properties and relationships to solve problems such as finding unknown sides and angles in diagrams (Applying Strategies, Reasoning)
- justify their solutions to problems by giving reasons using their own words (Reasoning, Communicating)
- recognise that area, length of matching sides and angle sizes are preserved in congruent figures (Reasoning)
- recognise that shape, angle size and the ratio of matching sides are preserved in similar figures (Reasoning)
- recognise that similar and congruent figures are used in specific designs, architecture and art work eg works by Escher, Vasarely and Mondrian; or landscaping in European formal gardens (Reflecting)
- find examples of similar and congruent figures embedded in designs from many cultures and historical periods (Reflecting)
- use dynamic geometry software to investigate the properties of geometrical figures
(Applying Strategies, Reasoning)


## Background Information

Similarity is linked with ratio in the Number strand and with map work in Geography.

Similar and congruent figures are embedded in a variety of designs (eg tapa cloth, Aboriginal designs, Indonesian ikat designs, Islamic designs, designs used in ancient Egypt and Persia, window lattice, woven mats and baskets).

## Language

The term 'corresponding' is often used in relation to congruent and similar figures to refer to angles or sides in the same position, but it also has a specific meaning when used to describe a pair of angles in relation to lines cut by a transversal. This syllabus has used 'matching' to describe angles and sides in the same position; however, the use of the word 'corresponding' is not incorrect.

The term 'superimpose' is used to describe the placement of one figure upon another in such a way that the parts of one
coincide with the parts of the other.

- determining the scale factor for a pair of circles
- calculating dimensions of similar figures using the enlargement or reduction factor
- choosing an appropriate scale in order to enlarge or reduce a diagram
- constructing scale drawings
- drawing similar figures using geometrical instruments


## Early Stage 1

## Position

## SGES1.3

Uses everyday language to describe position and give and follow simple directions

## Knowledge and Skills

## Students learn about

- giving and following simple directions to position an object or themselves
eg 'Put the blue teddy in the circle.'
- using everyday language to describe their position in relation to other objects eg 'I am sitting under the tree.'
- using everyday language to describe the position of an object in relation to themselves eg 'The table is behind me.'
- using everyday language to describe the position of an object in relation to another object eg 'The book is inside the box.'


## Key Ideas

Give and follow simple directions
Use everyday language to describe position

## Working Mathematically

## Students learn to

- participate in movement games involving turning and direction (Applying Strategies, Reflecting)
- follow directions to a point or place including in mazes, games and computer applications (Applying Strategies, Reflecting)
- direct simple computer-controlled toys and equipment to follow a path (Applying Strategies)


## Background Information

There are two main ideas for students at this Stage:

- following an instruction to position an object or themselves,
and
- describing the relative position of an object or themselves.

Many students may be able to describe the position of an object in relation to themselves, but not in relation to another object.

Stage 1

## Position

## SGS1. 3

Represents the position of objects using models and drawings and describes using everyday language

## Key Ideas

Represent the position of objects using models and drawings Describe the position of objects using everyday language, including 'left' and 'right'

## Working Mathematically

## Students learn to

- give or follow instructions to position objects in models and drawings eg 'Draw the bird between the two trees.' (Communicating)
- use a diagram to give simple directions (Applying Strategies, Communicating)
- give or follow simple directions using a diagram or description (Applying Strategies, Communicating)
- create a path using computer drawing tools (Applying Strategies, Reflecting)


## Background Information

Making models and drawing simple sketches of their models is the focus at this Stage. Students usually concentrate on the relative position of objects in their sketches. The relationship of size between objects is difficult and will be refined over time, leading to the development of scale drawings in later Stages. Accepting students' models and sketches is important.

Being able to describe the relative position of objects in a picture or diagram requires interpretation of a two-dimensional representation.

Stage 2

## Position

## SGS2. 3

Uses simple maps and grids to represent position and follow routes

## Key Ideas

Use simple maps and grids to represent position and follow routes
Determine the directions N, S, E and W; NE, NW, SE and SW, given one of the directions
Describe the location of an object on a simple map using coordinates or directions

## Knowledge and Skills

## Students learn about

- describing the location of an object using more than one descriptor eg 'The book is on the third shelf and second from the left.'
- using a key or legend to locate specific objects
- constructing simple maps and plans eg map of their bedroom
- using given directions to follow a route on a simple map
- drawing and describing a path or route on a simple map or plan
- using coordinates on simple maps to describe position eg 'The lion's cage is at B3.'
- plotting points at given coordinates
- using a compass to find North and hence East, South and West
- using an arrow to represent North on a map
- determining the directions $\mathrm{N}, \mathrm{S}, \mathrm{E}$ and W , given one of the directions
- using N, S, E and W to describe the location of an object on a simple map, given an arrow that represents North eg 'The treasure is east of the cave.'
- using a compass rose to indicate each of the key directions
eg

- determining the directions NE, NW, SE and SW, given one of the directions
- using NE, NW, SE and SW to describe the location of an object on a simple map, given a compass rose eg 'The treasure is north-east of the cave.'


## Working Mathematically

## Students learn to

- use and follow positional and directional language (Communicating)
- create simple shapes using computer software involving direction and angles (Applying Strategies)
- discuss the use of grids in the environment eg zoo map, map of shopping centre (Communicating, Reflecting)
- use computer software involving maps, position and paths (Applying Strategies)
- create a simple map or plan using computer paint, draw and graphics tools (Applying Strategies)
- use simple coordinates in games, including simulation software (Applying Strategies)
- interpret and use simple maps found in factual texts and on the Internet (Applying Strategies, Communicating)


## Background Information

Grids are used in many contexts to identify position. Students could create their own simple maps and, by drawing a grid over the map, they can then describe locations.

Students need to have experiences identifying North from a compass in their own environment and then determining the other three directions, East, West and South. This could be done in the playground before introducing students to using these directions on maps to describe the positions of various places. The four directions NE, NW, SE and SW could then be introduced to assist with descriptions of places that lie between N, S, E or W.

Stage 3

## Position

## SGS3. 3

Uses a variety of mapping skills

## Key Ideas

Interpret scales on maps and plans
Make simple calculations using scale

## Working Mathematically

## Students learn to

- use coordinates in simulation software and spreadsheets (Applying Strategies)
- interpret scales on maps and plans (Applying Strategies, Reflecting)
- give reasons for using a particular scale on a map or plan (Reasoning)
- recognising that the same location can be represented by maps or plans using different scales
- using scale to calculate the distance between two points on a map
- locating a place on a map which is a given direction from a town or landmark
eg locating a town that is north-east of Broken Hill
- drawing maps and plans from an aerial view


## Background Information

At this Stage, a range of mapping skills could be further developed that include the interpretation of scales and simple calculations to find the actual distance between locations on a map.

This topic links to Human Society and Its Environment (HSIE). These skills could be used to explore the sizes of other countries relative to Australia.

## Language

The word 'scale' has different meanings in different contexts. Scale could mean the enlargement or reduction factor for a drawing, the scale marked on a measuring device or a fish scale.

## General Principles for Planning, Programming, Assessing, Reporting and Evaluating

Planning, programming, assessing and reporting in Mathematics involve the consideration of the individual learning needs of all students and the creation of a learning environment that assists students to achieve the outcomes of the syllabus.
Students' achievement of the syllabus outcomes is the goal of planning, programming and assessing. The sequence of learning experiences that teachers provide should build on what students already know and should be designed to ensure that they progress through the Stages identified in the learning continuum. As students participate in a range of learning experiences in mathematics, teachers make judgements about what students know, what they can do and what they understand.
Evaluating is the process of making judgements about the effectiveness of school/faculty plans, teaching programs, policies and procedures.

## Assessment

## Standards

The Board of Studies K-10 curriculum framework is a standards-referenced framework that describes, through syllabuses and other documents, the expected learning outcomes for students.

Standards in the framework consist of two interrelated elements:

- outcomes and content in syllabuses showing what is to be learned
- descriptions of levels of achievement of that learning.

Exemplar tasks and student work samples help to elaborate standards.
Syllabus outcomes in mathematics contribute to a developmental sequence in which students are challenged to acquire new knowledge, skills and understanding.

The standards are typically written for two years of schooling and set high, but realistic, expectations of the quality of learning to be achieved by most students by the end of Years $2,4,6,8,10$ and 12 .

## Using standards to improve learning

Teachers will be able to use standards in mathematics as a reference point for planning teaching and learning programs, and for assessing and reporting student progress. Standards in mathematics will help teachers and students to set targets, monitor achievement, and as a result make changes to programs and strategies to support and improve each student's progress.

## Assessment for learning

Assessment for learning in mathematics is designed to enhance teaching and improve learning. It is assessment that gives students opportunities to produce the work that leads to development of their knowledge, skills and understanding. Assessment for learning involves teachers in deciding how and when to assess student achievement, as they plan the work students will do, using a range of appropriate assessment strategies including self-assessment and peer assessment.
Teachers of mathematics will provide students with opportunities in the context of everyday classroom activities, as well as planned assessment events, to demonstrate their learning.

In summary, assessment for learning:

- is an essential and integrated part of teaching and learning
- reflects a belief that all students can improve
- involves setting learning goals with students
- helps students know and recognise the standards they are aiming for
- involves students in self-assessment and peer assessment
- provides feedback that helps students understand the next steps in learning and plan how to achieve them
- involves teachers, students and parents reflecting on assessment data.


## Quality Assessment Practices

The following principles provide the criteria for judging the quality of assessment materials and practices.

## Assessment for learning:

- emphasises the interactions between learning and manageable assessment strategies that promote learning In practice, this means:
- teachers reflect on the purposes of assessment and on their assessment strategies
- assessment activities allow for demonstration of learning outcomes
- assessment is embedded in learning activities and informs the planning of future learning activities
- teachers use assessment to identify what a student can already do
- clearly expresses for the student and teacher the goals of the learning activity

In practice, this means:

- students understand the learning goals and the criteria that will be applied to judge the quality of their achievement
- students receive feedback that helps them make further progress
- reflects a view of learning in which assessment helps students learn better, rather than just achieve a better mark In practice, this means:
- teachers use tasks that assess, and therefore encourage, deeper learning
- feedback is given in a way that motivates the learner and helps students to understand that mistakes are a part of learning and can lead to improvement
- assessment is an integral component of the teaching-learning process rather than being a separate activity
- provides ways for students to use feedback from assessment

In practice, this means:

- feedback is directed to the achievement of standards and away from comparisons with peers
- feedback is clear and constructive about strengths and weaknesses
- feedback is individualised and linked to opportunities for improvement
- helps students take responsibility for their own learning

In practice, this means:

- assessment includes strategies for self-assessment and peer assessment emphasising the next steps needed for further learning
- is inclusive of all learners

In practice, this means:

- assessment against standards provides opportunities for all learners to achieve their best
- assessment activities are free of bias.


## Making judgements about student achievement

Assessment for learning in the Mathematics K-6 Syllabus is designed to give students opportunities to produce the work that leads to development of their knowledge, skills and understanding. It involves teachers in deciding how and when to assess student achievement, as they plan the work students will do, using a range of appropriate assessment strategies including self-assessment and peer assessment. Teachers of mathematics provide students with opportunities in the context of everyday classroom activities, as well as planned assessment events, to demonstrate their learning.

Gathered evidence can also be used for assessment of learning that takes place at key points in the learning cycle, such as the end of a year or Stage, when schools may wish to report differentially on the levels of skill, knowledge, skills and understanding achieved by students.

## Choosing Assessment Strategies

In Years K-10 Mathematics, assessment of student achievement should incorporate measures of students':

- ability to work mathematically
- knowledge, understanding and skills related to: Number; Patterns and Algebra; Data; Measurement; and Space and Geometry.

Students indicate their level of understanding and skill development in what they do, what they say, and what they write and draw. The most appropriate method or procedure for gathering assessment information is best decided by considering the purpose for which the information will be used, and the kind of performance that will provide the information. Consequently there is a variety of ways to gather assessment information in mathematics. Tasks given to students for the purpose of gathering assessment information include projects, investigations, oral reports or explanations, tests, and practical assignments. For example, practical tasks would often be an appropriate strategy for the assessment of achievement of outcomes for Measurement.

Teachers have the opportunity to observe and record aspects of students' learning in a range of situations. When students are working in groups, teachers are well placed to determine the extent of student interaction and participation. By listening to what students say - including their responses to questions or other input - teachers are able to collect many clues about students' existing understanding and attitudes. Through interviews (which may only be a few minutes in duration), teachers can collect specific information about the ways in which students think in certain situations. The students' responses to questions and comments will often reveal their levels of understanding, interests and attitudes. Records of such observations form valuable additions to information gained using other assessment strategies, and enhance teachers' judgement of their students' achievement of outcomes.

Consideration of students' journals or their comments on the process of gaining a solution to a problem can also be very enlightening for teachers and provide valuable insight into the extent of students' mathematical thinking.

Possible sources of information for assessment purposes include the following:

- samples of students' work
- explanation and demonstration to others
- questions posed by students
- practical tasks such as measurement activities
- investigations and/or projects
- students' oral and written reports
- short quizzes
- pen-and-paper tests
- comprehension and interpretation exercises
- student-produced worked examples
- teacher/student discussion or interviews
- observation of students during learning activities, including listening to students' use of language


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- observation of students' participation in a group activity
- consideration of students' portfolios
- students' plans for and records of their solutions of problems
- students' journals and comments on the process of their solutions.


## Reporting

Reporting is the process of providing information, both formally and informally, about the process of student achievement. Reports can be presented in a spoken or written form. The principals below underpin effective reporting.

- Reporting students' achievement has a number of purposes for a variety of audiences such as students, parents/caregivers, teachers, the school and the wider community.
- Reporting should provide a diagnosis of areas of strength and need, including those in which the students might be given additional support.
- Reporting information needs to be clear and appropriate to the audience.

Parents will want to know how their child is progressing in relation to:

- values and attitudes
- knowledge and understanding in and about mathematics and working mathematically
- skills and competence in using mathematics.

When reporting to parents, key features of the report should include:

- information about how the student is progressing
- suggestions of ways the parents can help at home to develop the child's confidence to take risks with mathematics.


## Evaluating

Evaluation is an ongoing process. Information for use in evaluation may be gathered through: student assessment; teachers' own reflection on their teaching practices; written records such as questionnaires, logs and diaries, submissions or records of meetings; and discussion with general staff members, teaching staff (including any specialist teachers involved), parents and other community members.

Teachers need to gather, organise and interpret information in order to make judgements about the effectiveness and appropriateness of:

- curriculum overviews and plans
- teaching programs
- teaching strategies
- assessment strategies
- resources
- staff development programs.


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## Indicators

Each outcome is accompanied by a sample set of indicators. An indicator is a statement of the behaviour that students might display as they work towards the achievement of a syllabus outcome.

Indicators are included in this syllabus to help exemplify the range of observable behaviours that contribute to the achievement of outcomes linked to the content. They can be used by teachers to monitor student progress within a Stage and to make on-balance judgements about the achievement of outcomes at the end of a Stage. Teachers may wish to develop their own indicators or modify the syllabus indicators, as there are numerous ways that students may demonstrate what they know and can do.

Indicators are not content. At the end of each list of indicators, there is a page reference to the relevant content section.

It is important that teachers use the content section for programming since this includes all of the key ideas as well as a comprehensive list of the knowledge and skills, and suggestions for the integration of Working Mathematically processes.

## Working Mathematically

## Questioning

Students ask questions in relation to mathematical situations and their mathematical experiences

## Early Stage 1

## WMES1.1

Asks questions that could be explored using mathematics in relation to Early Stage 1 content

The student, for example:

- asks questions, with guidance, that can be answered in part by sorting, placing in order, or counting
- asks questions involving counting numbers to at least 20 eg 'How many pencils are in the tin?', 'Who has more?'
- poses questions about situations using everyday language eg 'What colour hair do most people in our class have?'
- poses problems about sharing a collection of objects
- asks questions about how a repeating pattern can be copied or continued
- asks questions involving measurement in everyday situations eg 'Which book cover is bigger?', 'Who has the longest pencil?'
- asks questions related to time eg 'Is tomorrow Wednesday?', 'How long is it until lunchtime?', 'Does the clock say 3 o'clock?'
- asks and responds to questions that help identify a given shape

Working Mathematically embedded in all content

## Stage 1

## WMS1. 1

Asks questions that could be explored using mathematics in relation to Stage 1 content

The student, for example:

- poses questions that are similar to, or related to, a modelled question
- asks questions about quantities eg 'How many lollies will I need to fill the jar?'
- asks questions about situations eg 'Do I have enough time to finish my task before recess?'
- asks questions involving two- and three-digit numbers eg 'Why is 153 less than 163 ?'
- poses problems that can be solved using addition and subtraction, including those involving money
- questions if parts of a whole object, or collection of objects, are equal
- asks questions about informal units of measurement eg 'Will unifix cubes or longs be better to cover my book?'
- asks questions related to the size and mass of objects eg 'Why is this small wooden block heavier than this empty plastic bottle?'
- asks mathematical questions about stories eg 'Why did the boat sink when the mouse got in?'


## Working Mathematically

## Questioning

Students ask questions in relation to mathematical situations and their mathematical experiences

## Stage 2

## WMS2. 1

Asks questions that could be explored using mathematics in relation to Stage 2 content

The student, for example:

- asks questions that clarify a mathematical situation and enable progression towards a solution
- generates questions when planning an excursion eg 'How much will the bus cost?', 'How long will the bus journey take?'
- poses questions about a collection of items eg 'Is it possible to find one-eighth of this collection of objects?'
- poses problems based on number patterns
- poses questions that can be answered using the information from a table or graph
- poses a suitable question to be answered using a survey
- questions why two students may obtain different measurements for the same length, perimeter, area, volume, capacity or mass

Working Mathematically embedded in all content

## Stage 3

## WMS3. 1

Asks questions that could be explored using mathematics in relation to Stage 3 content

The student, for example:

- asks questions to extend mathematical tasks eg 'If we surveyed the whole school, would we get a similar result to that obtained from our class?'
- asks 'what if' questions eg 'What happens if we subtract a larger number from a smaller number using a calculator?'
- poses problems that can be solved using numbers of any size and more than one operation
- asks questions about how number patterns with one operation have been created and how they can be continued
- asks questions to determine a number pattern generated by the teacher or another student


## Working Mathematically embedded in all content

Stage 4

## WMS4. 1

Asks questions that could be explored using mathematics in relation to Stage 4 content

The student, for example:

- clarifies and refines mathematical questions to help understand or guide the investigation of a situation
- asks questions about numbers that do not terminate or recur when expressed as a decimal eg $\sqrt{13}$
- asks ‘what if' questions eg 'What happens to a graph if I add a constant to the equation?'
- poses problems that can be solved using Pythagoras' theorem

Working Mathematically embedded in all content

Working Mathematically

## Applying Strategies

Students develop, select and use a range of strategies, including the selection and use of appropriate technology, to explore and solve problems

## Early Stage 1

## WMES1.2

Uses objects, actions, imagery, technology and/or trial and error to explore mathematical problems

The student, for example:

- solves problems using strategies that include using objects, trial and error and acting it out
- compares groups of objects by one-to-one correspondence
- creates a shape using computer paint, draw or graphics tools
- compares the masses of two objects by hefting or using an equal arm balance
- uses a plank or board to determine which objects roll or slide
- uses beads threaded onto a string to make a 'three' pattern
- represents the information in a problem by drawing a picture or using objects
- uses direct comparison to determine which of two objects is longer, taller, wider, bigger, heavier or holds more
- explores mathematical concepts and answers questions by acting out a story

Working Mathematically embedded in all content

## Stage 1

## WMS1. 2

Uses objects, diagrams, imagery and technology to explore mathematical problems

The student, for example:

- solves problems using strategies that include drawing diagrams
- uses computer drawing tools to complete a design with one line of symmetry
- uses coins to work out what could be bought at the canteen for $\$ 1$
- solves problems that relate to their environment eg comparing the lengths of two fixed objects that cannot be aligned
- selects and uses appropriate informal units to measure an attribute
- imagines a group of objects and mentally takes away objects to work out how many are left
- uses a variety of strategies to solve addition and subtraction problems
- visualises and makes recently seen shapes
- uses simple graphics software to create a picture graph

Working Mathematically embedded in all content

Working Mathematically

## Applying Strategies

Students develop, select and use a range of strategies, including the selection and use of appropriate technology, to explore and solve problems

## Stage 2

## WMS2.2

Selects and uses appropriate mental or written strategies, or technology, to solve problems

The student, for example:

- solves problems using strategies that include creating patterns and constructing tables
- uses problem-solving strategies including those based on selecting key information and showing it in models, diagrams and lists
- uses a calculator to solve a number problem that has occurred in an everyday context
- uses efficient strategies for counting a large number of square centimetres
- uses simple graphing software to enter data and create a graph
- selects and uses the best measuring tool for a given task eg finding the dimensions of the netball court

Working Mathematically embedded in all content

## Stage 3

## WMS3. 2

Selects and applies appropriate problem-solving strategies, including technological applications, in undertaking investigations

The student, for example:

- solves problems using strategies that include working backwards and simplifying the problem
- uses problem-solving strategies including those based on selecting and organising key information in a systematic way
- uses a calculator to carry out an investigation eg explores procedures for multiplying decimal numbers by multiples of ten using a calculator
- plans a trip to a given destination using timetables, maps and street directories
- uses a computer database to organise information collected from a survey
- breaks a problem down into a series of simpler problems

Working Mathematically embedded in all content

## Stage 4

## WMS4. 2

Analyses a mathematical or real-life situation, solving problems using technology where appropriate

The student, for example:

- solves problems using strategies that include identifying and working on related problems
- solves simple probability problems arising in games
- determines a set of data that has a particular mean, median and mode
- uses appropriate technology to explore a mathematical situation

Working Mathematically embedded in all content

## Working Mathematically

## Communicating

Students develop and use appropriate language and representations to formulate and express mathematical ideas

## Early Stage 1

## WMES1.3

Describes mathematical situations using everyday language, actions, materials and informal recordings

The student, for example:

- talks about mathematical experiences
- talks about objects using descriptive language in reference to at least one attribute
- uses comparative language to describe a situation eg 'I think the pencil is longer than the scissors.'
- describes distances informally
- describes his/her position in relation to other objects
- draws a picture to show results
- manipulates and describes a hidden object using everyday language
- describes the sorting of objects using everyday language
- demonstrates how he/she determined which shape has the biggest area

Working Mathematically embedded in all content

## Stage 1

## WMS1.3

Describes mathematical situations and methods using everyday and some mathematical language, actions, materials, diagrams and symbols

The student, for example:

- describes a situation using own language (language of choice may be a language other than English)
- draws a diagram to show how an answer was obtained
- discusses with a partner various ways of moving around the classroom using directional language
- uses a variety of own recording strategies
- follows simple directions given a diagram or description
- describes strategies used to solve a problem using language, actions, materials and drawings
- represents information using a data display
- describes objects using mathematical names for shapes and features

Working Mathematically embedded in all content

## Working Mathematically

## Communicating

Students develop and use appropriate language and representations to formulate and express mathematical ideas

## Stage 2

## WMS2.3

Uses appropriate terminology to describe, and symbols to represent, mathematical ideas

The student, for example:

- uses a graph to compare student preferences
- writes a procedure to outline the method used to solve a problem
- uses a map or a grid to represent the best way to get from one location to another
- describes objects using mathematical names for shapes and features, and metric units for measurements
- discusses fairness of simple games involving chance
- describes how an attribute was estimated and measured
- discusses different methods for solving a given problem
- explains the mental strategy used to solve a problem
- uses mathematical symbols to represent elements of a problem eg 'How many weeks in a year?' could be represented by $365 \div 7$ or 7 ) 365

Working Mathematically embedded in all content

## Stage 3

## WMS3. 3

Describes and represents a mathematical situation in a variety of ways using mathematical terminology and some conventions

The student, for example:

- uses correct mathematical language to explain mathematical situations eg cross-section, circumference
- explains mathematical problems in terms of practical solutions eg 'An answer of 1.5 buses means that 2 buses will be needed.'
- uses a line graph to record and interpret information and to predict a solution to another problem
- explains the advantages of using a cube as a unit to measure volume and capacity
- interprets negative whole numbers in everyday situations eg temperatures that are below zero
- interprets information from the media, the environment and other sources that use large numbers

Working Mathematically embedded in all content

## Stage 4

## WMS4. 3

Uses mathematical terminology and notation, algebraic symbols, diagrams, text and tables to communicate mathematical ideas

The student, for example:

- describes a solution to a problem using mathematical terminology and algebraic symbols
- interprets and explains media reports and advertising that quote various statistics
- uses algebraic symbols and some conventions to express a generalisation eg $a+b=b+a$

Working Mathematically embedded in all content

## Working Mathematically

## Reasoning

Students develop and use processes for exploring relationships, checking solutions and giving reasons to support their conclusions

## Early Stage 1

## WMES1.4

Uses concrete materials and/or pictorial representations to support conclusions

The student, for example:

- checks answers by repeating the process eg ordering numbers
- refers to a drawing to demonstrate how an answer was obtained eg 'I drew ten sausages and crossed two off to show that eight were left.'
- explains the reason for halving an object in a particular way
- explains why the length of a piece of string remains unchanged if placed in a straight line or a curve
- gives reasons why he/she thinks one object will be longer, taller, wider, bigger, heavier or will hold more, than another
- explains why a collection of objects has been sorted in a particular way
- recognises when an error occurs in a pattern and explains what is wrong

Working Mathematically embedded in all content

## Stage 1

## WMS1.4

Supports conclusions by explaining or demonstrating how answers were obtained

The student, for example:

- explains why an answer is correct
- changes the answer if, when redoing the question, he/she believes it to be incorrect
- expresses a point of view about the correctness of an answer
- explains why parts of a whole are equal
- checks solutions to missing elements in patterns by repeating the process
- checks number sentences to see if they are true or false
- explains the appropriateness of a selected informal unit in measurement
- explains that if a smaller unit is used then more units are needed to measure eg 'More cups than ice cream containers are needed to fill the bucket.'
- gives reasons for placing a set of numbers in a particular order

Working Mathematically embedded in all content

## Working Mathematically

## Reasoning

Students develop and use processes for exploring relationships, checking solutions and giving reasons to support their conclusions

## Stage 2

## WMS2.4

Checks the accuracy of a statement and explains the reasoning used

The student, for example:

- checks solutions to problems and evaluates the method used
- checks the answer to a subtraction problem using addition
- checks the reasonableness of a solution to a problem by relating it to an original estimation
- compares the likelihood of outcomes in a simple chance experiment
- checks solutions to missing elements in patterns by repeating the process
- compares tables and graphs constructed from the same data to determine which is the most appropriate method of display
- explains why two students may obtain different results for the same measurement
- explains why a given angle is, or is not, a right angle
- uses an alternative method to confirm an answer

Working Mathematically embedded in all content

## Stage 3

## WMS3. 4

Gives a valid reason for supporting one possible solution over another

The student, for example:

- checks for accuracy at each stage of the solution to a problem
- uses a level of accuracy appropriate in the context of the problem eg height in cm not mm
- argues the case for an answer
- justifies the methods used to find another solution
- explains and gives reasons why particular results were obtained
- checks that an answer makes sense in the context of the problem
- justifies the choice of a particular rule for the values in a table
- gives reasons for a particular scale on a map
- corrects answers and explains where his/her thinking or execution was incorrect
- works backwards to check a solution

Working Mathematically embedded in all content

## Stage 4

## WMS4. 4

Identifies relationships and the strengths and weaknesses of different strategies and solutions, giving reasons

The student, for example:

- compares the strengths and weaknesses of different forms of data display
- sorts and classifies equations of linear relationships into groups to demonstrate similarities and differences
- explains why a particular relationship or rule for a given pattern is better than another

Working Mathematically embedded in all content

## Reflecting

Students reflect on their experiences and critical understanding to make connections with, and generalisations about, existing knowledge and understanding

## Early Stage 1

## WMES1.5

Links mathematical ideas and makes connections with, and generalisations about, existing knowledge and understanding in relation to Early Stage 1 content

The student, for example:

- identifies the use of numbers and shapes in everyday experiences eg 'I see lots of squares in the classroom.'
- gives examples of where numbers can be seen eg books, telephones, computer keyboards
- recognises and explains which three-dimensional objects pack and stack easily
- predicts which object would be heavier than, lighter than, or have about the same mass as another object
- makes an observation that can lead to a generalisation eg 'You are taller than me because I have to look up when I stand beside you.'
- makes generalisations about combining groups eg 'When I put two groups of counters together, I always end up with more counters.'
- links shapes and numbers when creating repeating patterns

Working Mathematically embedded in all content

## Stage 1

## WMS1.5

Links mathematical ideas and makes connections with, and generalisations about, existing knowledge and understanding in relation to Stage 1 content

The student, for example:

- identifies and appreciates some of the ways in which numbers and measurements are used in people's lives eg house and telephone numbers, daily activities related to time
- gives examples of where measurements are used to describe people or objects
- links experiences with symmetry to descriptions of a half of an object or picture
- determines which shapes covered better when measuring the surface of the table
- recognises the use of half in everyday situations
- makes an observation that can lead to a generalisation eg 'These squares fit together better than the circles.'
- makes generalisations about number relationships eg 'When I count by tens, the last digit is always the same.'
- makes generalisations about measuring eg 'When measuring a longer distance, I need to use a longer informal unit.'
- recognises that models with different shapes may have the same volume

Working Mathematically embedded in all content

## Working Mathematically

## Reflecting

Students reflect on their experiences and critical understanding to make connections with, and generalisations about, existing knowledge and understanding

## Stage 2

## WMS2.5

Links mathematical ideas and makes connections with, and generalisations about, existing knowledge and understanding in relation to Stage 2 content

The student, for example:

- identifies and describes the use of mathematics in everyday contexts
- gives examples of where prisms and pyramids are used in packing materials and designing buildings
- applies an understanding of equally likely outcomes in games and other simple situations involving random generators eg dice, coins, spinners
- recognises that objects with a mass of one kilogram can be a variety of shapes
- compares features of three-dimensional objects and twodimensional shapes
- makes generalisations about creating fractional parts of collections of objects eg 'To make quarters, I just need to divide the objects into four groups.'
- links finding lines of symmetry of a shape with creating fractional parts of a whole

Working Mathematically embedded in all content

## Stage 3

## WMS3.5

Links mathematical ideas and makes connections with, and generalisations about, existing knowledge and understanding in relation to Stage 3 content

The student, for example:

- explains some ways that mathematics is used, or has been used, to represent, describe and explain our world
- makes generalisations about number relationships
- gives examples of where fractions and percentages are used in real contexts
- explains the difference between area and perimeter
- links multiplication with finding the area of a rectangle

Working Mathematically
embedded in all content

## Stage 4

WMS4.5
Links mathematical ideas, and makes connections with and generalisations about, existing knowledge and understanding in relation to Stage 4 content

The student, for example:

- makes links between area, surface area and volume of particular solids
- makes links between the graphical representations of straight lines and their equations by matching them
- identifies and explains the use of mathematics in historical and cultural contexts

Working Mathematically embedded in all content

## Number

## Whole Numbers

Students develop a sense of the relative size of whole numbers and the role of place value in their representation

## Early Stage 1

## NES1.1

Counts to 30, and orders, reads and represents numbers in the range 0 to 20

The student, for example:

- counts forwards to 30 , from a given number
- counts backwards from a given number, in the range 0 to 20
- names the number before and after a given number
- demonstrates one-to-one correspondence when counting to 20
- reads and records numbers up to 20 , including 0
- makes groups of objects up to 20
- matches numerals to the number of objects up to 20
- orders a set of numbers up to 20 from smallest to largest
- names instantly the number represented by an arrangement of dots on a standard die
- estimates the number of objects in a group and counts to check
- uses the ordinal names 'first' to 'tenth'

Related syllabus content on p 41

## Stage 1

## NS1.1

Counts, orders, reads and represents two- and three-digit numbers

The student, for example:

- counts forwards or backwards from a given two-digit number
- names the number before and after a given three-digit number
- reads, writes and says two- and three-digit numbers
- states the place value of digits in a three-digit number
- makes the largest or smallest number given any three digits
- orders a set of two- and three-digit numbers
- uses the terms 'more than' and 'less than' when comparing numbers
- uses ordinal names 'first' to 'thirty-first' on a calendar
- counts forwards or backwards by twos, fives or tens
- represents two- and three-digit numbers using materials eg bundles of popsticks
- orders a collection of notes or coins according to face value

Related syllabus content on pp 42-43

## Number

## Whole Numbers

Students develop a sense of the relative size of whole numbers and the role of place value in their representation

## Stage 2

## NS2. 1

Counts, orders, reads and records numbers up to four digits

The student, for example:

- names the number before and after a given four-digit number
- reads, writes and says three- and four-digit numbers
- states the place value of digits in a four-digit number
- makes the largest and smallest number given any four digits
- places a set of three- and four-digit numbers in ascending or descending order
- uses the symbols for 'is less than' $(<)$ and 'is greater than' ( $>$ ) to show the relationship between two numbers
- counts forwards or backwards from any four-digit number by tens or hundreds
- records three- and four-digit numbers using expanded notation eg $5429=5000+400+20+9$
- rounds numbers to the nearest ten, hundred or thousand when estimating

Related syllabus content on p 44

## Stage 3

## NS3. 1

Orders, reads and writes numbers of any size

The student, for example:

- reads, writes and says large numbers
- writes a number presented orally
- makes the second largest or second smallest number, given any four digits
- explains the place value of any digit in a number
- places a set of large numbers in ascending or descending order
- records large numbers using expanded notation eg $59675=50000+9000+600+70+5$
- rounds numbers to the nearest ten thousand when estimating eg 92000 rounds to 90000
- matches different abbreviations of numbers used in everyday contexts eg \$350 K represents \$350 000
- orders a set of single digit numbers, including some negative numbers, on a number line

Related syllabus content on p 45

## Number

## Addition and Subtraction

Students develop facility with number facts and computation with progressively larger numbers in addition and subtraction and an appreciation of the relationship between those facts

## Early Stage 1

## NES1. 2

Combines, separates and compares collections of objects, describes using everyday language and records using informal methods

The student, for example:

- combines two or more groups of objects to model addition
- separates and takes part of a group of objects away to model subtraction
- compares two groups of objects and describes 'how many more'
- joins two groups of objects and states the number altogether
- uses concrete materials to model different combinations to 10 eg using a ten-frame and counters
- describes the action of combining using everyday language such as 'makes', 'join' and 'together'
- takes part of a group of objects away and states the number of objects remaining
- describes the action of subtraction using everyday language such as 'take away'
- uses concrete materials, including fingers, to solve simple addition and subtraction problems
- records addition and subtraction informally using drawings, numerals and words

Related syllabus content on p 46

## Stage 1

## NS1. 2

Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and twodigit numbers

The student, for example:

- represents subtraction as the difference between two numbers
- creates simple addition and subtraction stories and picture problems
- records number sentences using the symbols,+- and $=$
- recalls addition and subtraction facts for numbers to 20
- uses two or more different strategies to solve an addition or subtraction problem
- explains how an answer to an addition or subtraction problem was obtained
- counts on from the larger number to find the total of two numbers
- counts on or back to find the difference between two numbers
- bridges to ten to assist addition eg $17+5 ; 17$ and 3 is 20 and add 2 more
- recognises related addition and subtraction number sentences eg $8+2$ is 10 so $10-2$ is 8
- uses popsticks to perform addition and subtraction of twodigit numbers with trading
- uses an empty number line to record strategies used to solve addition or subtraction problems
- performs simple calculations with money

Related syllabus content
on pp 47-48

## Number

## Addition and Subtraction

Students develop facility with number facts and computation with progressively larger numbers in addition and subtraction and an appreciation of the relationship between those facts

## Stage 2

## NS2. 2

Uses mental and written strategies for addition and subtraction involving two-, three- and four-digit numbers

The student, for example:

- uses patterns to extend number facts eg $5-2=3$, so $500-200$ is 300
- explains and records methods for adding and subtracting
- uses a split strategy for addition or subtraction
- uses an empty number line and jump strategies to represent solutions to addition and subtraction problems involving three- or four-digit numbers
- adds or subtracts two numbers, with and without trading, using concrete materials
- uses the formal written algorithm to solve addition or subtraction problems
- uses a calculator to solve addition and subtraction problems that include larger numbers contained in a problem context


## Related syllabus content

 on pp 49-50
## Stage 3

## NS3. 2

Selects and applies appropriate strategies for addition and subtraction with counting numbers of any size

The student, for example:

- chooses appropriately between mental, written and calculator methods for addition and subtraction problems
- gives reasons why a calculator was useful when solving a problem
- uses estimation to check solutions to addition and subtraction problems
- uses the formal written algorithm to solve addition and subtraction problems involving counting numbers of any size
- uses addition to check answers to subtraction problems
- adds numbers with different numbers of digits eg $42000+5123+246$

Related syllabus content on p 51

## Number

## Multiplication and Division

Students develop facility with number facts and computation with progressively larger numbers in multiplication and division and an appreciation of the relationship between those facts

## Early Stage 1 <br> Stage 1

## NES1.3

Groups, shares and counts collections of objects, describes using everyday language and records using informal methods

The student, for example:

- uses the term 'group' to describe a collection of objects
- uses the term 'sharing' to describe the distribution of a collection of objects
- uses concrete materials to solve grouping or sharing problems
- models and describes equal groups and equal rows
- recognises an unequal group and an unequal row
- labels the number of objects in a group or row
- records grouping and sharing informally using pictures, numerals and words

Related syllabus content on p 52

## NS1. 3

Uses a range of mental strategies and concrete materials for multiplication and division

The student, for example:

- counts by ones, twos, fives or tens
- describes collections of objects as 'rows of' or 'groups of'
- uses an array to model multiplication problems
- uses counting strategies to find the total number of objects eg rhythmic counting, repeated addition
- shares a collection of objects into equal groups to model division
- models division as repeated subtraction
- uses a number line or hundreds chart to solve multiplication and division problems
- recognises and names the symbols $\times, \div$ and $=$

Related syllabus content on p 53

## Number

## Multiplication and Division

Students develop facility with number facts and computation with progressively larger numbers in multiplication and division and an appreciation of the relationship between those facts

## Stage 2

## NS2. 3

Uses mental and informal written strategies for multiplication and division

The student, for example:

- uses mental strategies to recall multiplication facts to $10 \times 10$
- uses multiplication facts to work out division facts
- explains the relationship between multiplication facts eg explains how the 3 and 6 times tables are related
- uses mental strategies to divide a two-digit number by a one-digit number
- describes and records methods used to solve a multiplication or division problem
- identifies multiples and factors for a given number
- uses mental strategies to multiply a one-digit number by a multiple of 10
- uses mental strategies to multiply a two-digit number by a one-digit number
- explains and records remainders to division problems eg $17 \div 4=4$ remainder 1

Related syllabus content on pp 54-55

## Stage 3

## NS3. 3

Selects and applies appropriate strategies for multiplication and division

The student, for example:

- selects appropriate mental, written or calculator strategies to solve multiplication and division problems
- records a remainder as a fraction or decimal, where appropriate eg $25 \div 4=6 \frac{1}{4}$ or 6.25
- multiplies a three- or four-digit number by a one-digit number using a mental or written strategy
- multiplies a three-digit number by a two-digit number using the extended form of the formal written algorithm
- divides a number with three or more digits by a single divisor
- divides a number with three or more digits by a multiple of ten
- calculates solutions to problems involving mixed operations eg $5 \times 4+7=27$
- identifies prime and composite numbers from a group of mixed numbers


## Related syllabus content on p 56

## Stage 4

## Operations with Whole Numbers

## NS4. 1

Recognises the properties of special groups of whole numbers and applies a range of strategies to aid computation

The student, for example:

- describes the link between squares and square roots
- expresses a number as a product of its prime factors
- expresses a number as a product of its prime factors using index notation
- uses diagrams to represent figurate numbers
- uses mental or written strategies to aid computation

Related syllabus content on pp 57-58

## Integers

## NS4. 2

Compares, orders and calculates with integers

The student, for example:

- explains, in words, number sentences involving integers
- plots directed numbers on a number line
- explains why -8 is less than -3
- orders a set of integers
- adds or subtracts directed numbers
- multiplies or divides directed numbers


## Number

## Fractions and Decimals

Students develop an understanding of the parts of a whole, and the relationships between the different representations of fractions

## Early Stage $1 \quad$ Stage 1

## NES1.4

Describes halves, encountered in everyday contexts, as two equal parts of an object

The student, for example:

- shares an object by dividing it into two equal parts
- describes parts of an object using the term 'half'
- explains that halves are two equal parts
- recognises when two parts are not halves
- uses drawings to record a half of an object eg draws a pizza cut in half

Related syllabus content on p 60

## NS1. 4

Describes and models halves and quarters, of objects and collections, occurring in everyday situations

The student, for example:

- models and describes a half or a quarter of an object
- models and describes a half or a quarter of a collection of objects
- models and describes the dividing of a collection of objects into quarters
- uses fraction language in a variety of everyday contexts eg 'the half-hour'
- recognises when four parts are not quarters
- uses fraction notation for half $\left(\frac{1}{2}\right)$ and quarter $\left(\frac{1}{4}\right)$


## Number

## Fractions and Decimals

Students develop an understanding of the parts of a whole, and the relationships between the different representations of fractions

## Stage 2

## NS2.4

Models, compares and represents commonly used fractions and decimals, adds and subtracts decimals to two decimal places, and interprets everyday percentages

The student, for example:

- compares and orders fractions with the same denominator
- renames fractions where the numerator and denominator are the same as 1 eg $\frac{4}{4}=1$
- interprets the numerator and denominator of a fraction eg ' $\frac{3}{8}$ means 3 equal parts of 8 '
- models fifths and tenths of an object or collection of objects
- expresses whole numbers as decimals
- interprets decimal notation for tenths or hundredths
- adds or subtracts two decimal numbers with two decimal places
- finds equivalence between halves, quarters and eighths of an object or collection of objects
- rounds a number with one or two decimal places to the nearest whole number


## Related syllabus content

 on pp 62-63
## Stage 3

## NS3.4

Compares, orders and calculates with decimals, simple fractions and simple percentages

The student, for example:

- finds equivalent fractions using a diagram, number line or mental strategy
- expresses improper fractions as mixed numerals
- subtracts a unit fraction from a whole number
- adds or subtracts fractions that have the same denominator
- adds or subtracts decimal numbers that have a different number of decimal places
- multiplies or divides decimal numbers by single-digit numbers
- adds or subtracts simple fractions where one denominator is a multiple of the other

$$
\operatorname{eg} \frac{2}{3}+\frac{1}{6}=\frac{4}{6}+\frac{1}{6}=\frac{5}{6}
$$

- multiplies simple fractions by whole numbers
- calculates simple percentages of quantities

Related syllabus content on pp 64-65

## Stage 4

## Fractions, Decimals and Percentages

## NS4. 3

Operates with fractions, decimals, percentages, ratios and rates

The student, for example:

- adds or subtracts fractions using written methods
- expresses improper fractions as mixed numerals
- multiplies or divides mixed numerals
- adds, subtracts, multiplies and divides decimals
- expresses fractions as decimals or percentages
- increases or decreases a quantity by a given percentage
- uses ratio to compare quantities of the same type
- calculates speed given distance and time

Related syllabus content
on pp 66-67

## Number

## Chance

Students develop an understanding of the application of chance in everyday situations and an appreciation of the difference between theoretical and experimental probabilities

## Early Stage 1

No outcome at this Stage

## Stage 1

## NS1.5

Recognises and describes the element of chance in everyday events

The student, for example:

- uses the language of chance (eg 'will happen', 'might happen', 'might not happen') to describe everyday events
- describes the element of chance in familiar activities eg 'I might play with my friend after school.'
- describes familiar events as being possible or impossible
- compares two familiar events and explains which is more likely to happen

Related syllabus content on p 68

## Number

## Chance

Students develop an understanding of the application of chance in everyday situations and an appreciation of the difference between theoretical and experimental probabilities

## Stage 2

## NS2.5

Describes and compares chance events in social and experimental contexts

The student, for example:

- lists all the possible outcomes in a simple chance situation eg 'heads', 'tails' if a coin is tossed
- describes events as being certain or uncertain
- compares familiar events and describes them as being equally likely, more likely or less likely to occur
- predicts possible outcomes in a simple chance experiment eg 'You are more likely to draw out a blue ball because there are more blue than red in the bag.'
- explains the differences between expected results and actual results in a simple chance experiment
- conducts simple experiments using coins, dice or spinners and records the results


## Related syllabus content

on p 69

## Stage 3

## NS3.5

Orders the likelihood of simple events on a number line from zero to one

The student, for example:

- uses data to order chance events from least likely to most likely eg roll two dice twenty times and order the results according to how many times each total is obtained
- orders commonly used chance words on a number line between zero (impossible) and one (certain)
- assigns a numerical value to the likelihood of an event occurring eg there is a $50 \%$ chance
- describes the likelihood of an event occurring as being more or less than a half

Related syllabus content on p 70

## Stage 4

## Probability

NS4.4
Solves probability problems involving simple events
The student, for example:

- lists all possible outcomes of a simple event
- expresses the probability of a particular outcome as a fraction between 0 and 1
- describes the complement of an event
- finds the probability of a complementary event

Related syllabus content on p 71

## Patterns and Algebra

Students develop skills in creating, describing and recording number patterns as well as an understanding of the relationships between numbers

## Early Stage 1 <br> Stage 1

## PAES1. 1

Recognises, describes, creates and continues repeating patterns and number patterns that increase or decrease

The student, for example:

- copies and continues a repeating pattern made using sounds or actions
- recognises a repeating pattern
- continues a repeating pattern made from shapes, objects or pictures
- creates a repeating pattern using shapes, objects or pictures eg $\nabla, \Delta, \bullet, \nabla, \Delta$,
- describes a repeating pattern made from shapes by referring to the names of the shapes or their attributes
- describes a repeating pattern in terms of a 'number pattern' eg $\mathrm{O}, \nabla, \mathrm{O}, \nabla, \mathrm{O}, \nabla$ is a 'two' pattern
- creates a new repeating pattern that is similar to a given repeating pattern
- continues a simple number pattern that increases or decreases and explains how this was achieved
- uses the term 'is the same as' to express equality of groups

Related syllabus content on p 73

## PAS1. 1

Creates, represents and continues a variety of number patterns, supplies missing elements in a pattern and builds number relationships

The student, for example:

- identifies patterns when counting by ones, twos, fives or tens
- supplies the next number in an increasing or decreasing pattern and describes how it was determined
- creates and describes simple number patterns that increase or decrease
- determines a missing number in a number pattern and describes how it was determined
- creates number sentences to record equivalent number relationships eg $5+2=4+3$
- recognises patterns that can be created by recording all possible combinations for a given number
- identifies and describes the relationship between addition and subtraction facts eg $3+5=8$; hence $8-5=3$ and $8-3=5$

Related syllabus content on p 74

## Patterns and Algebra

Students develop skills in creating, describing and recording number patterns as well as an understanding of the relationships between numbers

## Stage 2

## PAS2.1

Generates, describes and records number patterns using a variety of strategies and completes simple number sentences by calculating missing values

The student, for example:

- identifies and records number patterns when counting forwards by threes, fours, fives, sevens, eights or nines
- creates a variety of number patterns using whole numbers, fractions or decimals
- uses the equals sign to record equivalent number relationships eg $4 \times 3=6 \times 2$
- recognises and describes patterns in multiplication facts to $10 \times 10$
- forms arrays using materials, to demonstrate multiplication patterns and relationships
eg $3 \times 5=15$

- explains the relationship between multiplication facts eg explains how the 3 and 6 times tables are related
- relates multiplication and division facts eg $6 \times 4=24$; so $24 \div 4=6$ and $24 \div 6=4$
- completes number sentences involving one operation by calculating missing values eg Find $\square$ so that $5+\square=13$

Related syllabus content on p 75

## Stage 3

## PAS3.1a

Records, analyses and describes geometric and number patterns that involve one operation using tables and words

The student, for example:

- builds a simple geometric pattern using materials
- completes a table of values for a geometric pattern or a number pattern
- calculates the value of a missing number in a table of values and explains how it was determined
- records a description of a number pattern using words
- determines a rule, in words, to describe the pattern presented in a table
- uses the rule for a pattern to calculate the corresponding value for a larger number

> Related syllabus content on p 76

PAS3.1b
Constructs, verifies and completes number sentences involving the four operations with a variety of numbers

The student, for example:

- completes number sentences that involve more than one operation by calculating missing values
- completes number sentences involving fractions or decimals eg Find $\square$ so that $7 \times \square=7.7$
- constructs a number sentence to match a problem that is presented in words and that requires finding an unknown
- checks a solution to a number sentence by substituting into the original question
- uses inverse operations to assist with the solution of a number sentence eg Find $\square$ so that $125 \div 5=\square$ becomes find $\square$ so that $\square \times 5=125$

Related syllabus content on p 77

## Patterns and Algebra

## Stage 4

## Algebraic Techniques <br> PAS4. 1

Uses letters to represent numbers and translates between words and algebraic symbols

The student, for example:

- translates from a word statement to an algebraic statement
- translates an algebraic statement into words
- models algebraic expressions using cups and counters
- uses cups and counters to add and subtract simple algebraic expressions

Related syllabus content on p 78

## Number Patterns

## PAS4. 2

Creates, records, analyses and generalises number patterns using words and algebraic symbols in a variety of ways

The student, for example:

- records a number pattern to describe a geometric pattern
- completes a table of values for a number pattern and explains how the answers were determined
- describes a number pattern using algebraic symbols
- calculates the corresponding values for larger numbers in a table of values

Related syllabus content on pp 79-80

## Patterns and Algebra

## Stage 4

## Algebraic Techniques <br> PAS4. 3

Uses the algebraic symbol system to simplify, expand and factorise simple algebraic expressions

The student, for example:

- simplifies algebraic expressions using standard conventions
- describes what is wrong with incorrect expansions eg what is wrong with $5(a+7)=5 a+7$ ?
- factorises expressions by identifying a common factor eg $x^{2}-x y=x(x-y)$
- generates a number pattern by substituting several values into an expression eg $4 y-2$ leads to the pattern $2,6,10$, $14, \ldots$

Related syllabus content on p 81

## Algebraic Techniques <br> PAS4. 4

Uses algebraic techniques to solve linear equations and simple inequalities

The student, for example:

- chooses and justifies a correct solution to an equation from a given set
- solves an equation by using algebraic methods
- substitutes into a given formula to find the value of the subject
- solves simple inequalities and graphs solutions on a number line

Related syllabus content on p 82

## Linear Relationships

PAS4.5
Graphs and interprets linear relationships on the number plane

The student, for example:

- names the coordinates of the origin and other points that lie on the $x$ and $y$ axes
- reads, plots and names ordered pairs from a number plane diagram
- plots points on the number plane from a table of values
- gives the coordinates of points that lie on a line drawn on the number plane

Related syllabus content
on p 83

## Data

Students inform their inquiries through gathering, organising, tabulating and graphing data

## Early Stage 1 <br> Stage 1

## DES1.1

Represents and interprets data displays made from objects and pictures

## The student, for example:

- sorts objects into groups according to a characteristic eg sorts lunch boxes according to colour
- organises a group of similar objects into rows or columns
- compares groups by counting
- uses a picture of an object to represent the object in a data display
- organises actual objects or pictures of the objects into a data display
- describes information presented in a data display eg 'I can see that there are more red lunch boxes.'
- interprets information presented in a data display to answer questions eg 'Most people in our class have brown eyes.'

Related syllabus content on p 85

## DS1. 1

Gathers and organises data, displays data using column and picture graphs, and interprets the results

The student, for example:

- poses a suitable question that can be answered by gathering and displaying data
- uses concrete materials, tally marks or symbols to keep track of collected data
- displays data using a symbol to represent data eg using a coloured square to represent each fruit
- displays data using an object to represent data eg using a block to represent each car
- uses a baseline and equal spacing when representing data in a display
- uses same-sized symbols when representing data
- displays data using a column graph or a picture graph
- interprets information presented in a given picture graph or column graph

Related syllabus content
on p 86

## Data

Students inform their inquiries through gathering, organising, tabulating and graphing data

## Stage 2

## Stage 3

## DS2. 1

Gathers and organises data, displays data using tables and graphs, and interprets the results

## The student, for example:

- poses a suitable question to be answered using a survey
- creates a simple table to organise data
- constructs a column graph or a picture graph on grid paper using one-to-one correspondence
- marks equal spaces on each axis, labels axes and names a column or picture graph
- interprets information presented in a given column graph or picture graph
- represents the same data in a table, a column graph and a picture graph
- creates a two-way table to organise data
- interprets information presented in a table

Related syllabus content on p 87

## DS3. 1

Displays and interprets data in graphs with scales of many-toone correspondence

The student, for example:

- finds the mean for a small set of data
- determines a suitable scale for data on a picture, column or line graph
- draws a picture graph where one picture or symbol represents more than one item eg $\boldsymbol{\bullet} 100$
- interprets graphs using the scale to make generalisations about data
- draws a line graph to represent data that demonstrates a continuous change eg hourly temperature
- names the category represented by each section in a divided bar graph or sector (pie) graph

Related syllabus content on p 88

## Stage 4

## Data Representation

## DS4. 1

Constructs, reads and interprets graphs, tables, charts and statistical information

The student, for example:

- displays information using a line graph
- displays information using a sector graph
- interprets information from a line graph
- interprets information from a conversion graph
- constructs a frequency distribution table for data collected from a survey
- draws a histogram from data presented in a frequency table
- constructs a dot plot for a small number of data points
- displays data in a stem-and-leaf plot, choosing an appropriate stem for the data

Related syllabus content on p 89

## Data Analysis and Evaluation <br> DS4. 2

Collects statistical data using either a census or a sample and analyses data using measures of location and range

The student, for example:

- determines whether it would be appropriate to collect data from a whole population or a sample to answer a particular question
- finds the mean, range, median and mode of a set of data presented in a frequency distribution table
- uses a spreadsheet to tabulate data and determine measures of location
- compares two sets of data (eg pulse rates before and after exercise) using a back-to-back stem-and-leaf plot
- identifies and comments on the bias of a given sample
- uses a random number generator on a calculator to select a sample

Related syllabus content
on p 90

## Measurement

## Length

Students distinguish the attribute of length and use informal and metric units for measurement

## Early Stage 1

## MES1. 1

Describes length and distance using everyday language and compares lengths using direct comparison

The student, for example:

- sorts objects into groups of long and short objects
- uses everyday language to describe length eg long, short, high, tall, low, the same
- describes an object as being shorter, longer, wider, deeper, thicker or thinner than another object
- describes distance using terms such as near, far, nearer, further and closer
- compares the lengths of two objects by placing the objects side-by-side and aligning the ends
- identifies an object that is longer or shorter than another object
- straightens a curved or bent length of material to check if two lengths are the same
- records length comparisons by drawing, tracing, or cutting and pasting


## Related syllabus content

 on p 92
## Stage 1

## MS1.1

Estimates, measures, compares and records lengths and distances using informal units, metres and centimetres

The student, for example:

- measures the length of an object by placing informal units end-to-end without gaps or overlaps
- estimates the number of units required to measure length or distance
- counts units to compare and order the length of two or more objects
- selects and uses an appropriate informal unit for measuring length eg uses paper clips instead of popsticks to measure a pencil
- describes and records length as the number and type of units used eg six paper clips long
- uses the abbreviation for metre (m) and centimetre (cm)
- estimates and measures lengths and distances to the nearest metre or half-metre
- classifies the lengths of objects as being more than, less than or about the same as a metre
- measures length using a 10 cm length, with 1 cm markings, as a measuring device

Related syllabus content
on p 93

## Measurement

## Length

Students distinguish the attribute of length and use informal and metric units for measurement

## Stage 2

## MS2. 1

Estimates, measures, compares and records lengths, distances and perimeters in metres, centimetres and millimetres

The student, for example:

- records lengths or distances using metres, centimetres and/or millimetres eg $1 \mathrm{~m} 25 \mathrm{~cm}, 5 \mathrm{~cm} 3 \mathrm{~mm}$
- gives examples of situations where a unit smaller than the centimetre is needed for measurement
- estimates, measures and compares the lengths of objects in metres, centimetres and millimetres
- estimates, measures and compares the distances between two objects in metres, centimetres and millimetres
- uses the abbreviation for millimetre ( mm )
- records lengths or distances using decimal notation to two decimal places eg 1.23 m
- uses the term 'perimeter' to describe the total distance around a shape
- estimates and measures the perimeter of two-dimensional shapes
- reads and interprets calibrations on measuring devices eg ruler, measuring tape

Related syllabus content on p 94

## Stage 3

## MS3. 1

Selects and uses the appropriate unit and device to measure lengths, distances and perimeters

The student, for example:

- gives examples of situations where a longer unit than the metre is needed for measurement
- measures a kilometre and half-kilometre
- converts between units when comparing lengths and distances eg metres and kilometres, centimetres and metres
- records lengths or distances using decimal notation to three decimal places eg 2.753 km
- interprets symbols used to record speed in kilometres
- selects and uses the appropriate measuring device to measure lengths, distances or perimeters
- selects and uses the appropriate unit to record lengths, distances or perimeters
- measures the perimeter of a large area
- estimates, measures and compares the perimeters of squares, rectangles and triangles
- explains that the perimeters of squares, rectangles and triangles can be found by finding the sum of the side lengths


## Measurement

## Area

Students distinguish the attribute of area and use informal and metric units for measurement

## Early Stage 1 <br> Stage 1

## MES1. 2

Describes area using everyday language and compares areas using direct comparison

The student, for example:

- covers a surface completely with smaller shapes
- makes a closed shape and describes the area of the shape
- uses everyday language to describe area eg surface, inside, outside
- describes an area as being bigger than, smaller than or the same as another area
- compares area by placing one area on top of another
- records area comparisons informally by drawing, tracing, or cutting and pasting

Related syllabus content on p 96

## MS1. 2

Estimates, measures, compares and records areas using informal units

The student, for example:

- measures area by placing identical informal units in rows or columns without gaps or overlaps
- estimates the number of informal units needed to measure area
- counts and records the number of units used and describes the part left over
- compares and orders two or more areas using informal units
- compares the areas of two surfaces which cannot be moved or superimposed
- chooses appropriate informal units to measure area eg those that tessellate
- describes the same area in terms of different-sized units used eg 'It took 10 tiles but only 4 books to cover the surface.'

Related syllabus content on p 97

## Measurement

## Area

Students distinguish the attribute of area and use informal and metric units for measurement

## Stage 2

## MS2. 2

Estimates, measures, compares and records the areas of surfaces in square centimetres and square metres

The student, for example:

- identifies areas that are less than, greater than or about the same as 100 square centimetres or 1 square metre
- estimates, measures and records the size of a small area in square centimetres
- measures and compares small areas using a squarecentimetre grid overlay
- constructs a square metre
- estimates the number of square metres in a given area
- measures and records an area using a square metre
- identifies areas that are less than, more than or about the same as a square metre
- records area using the abbreviations for square metres $\left(\mathrm{m}^{2}\right)$ and square centimetres $\left(\mathrm{cm}^{2}\right)$

Related syllabus content on p 98

## Stage 3

## MS3. 2

Selects and uses the appropriate unit to calculate area, including the area of squares, rectangles and triangles

The student, for example:

- explains the need for a unit larger than a square metre
- gives examples of where square kilometres are used for measuring area eg suburbs, towns
- explains what can be appropriately measured in hectares and why square metres would not be used
- recognises the relationship between square metres and hectares
- selects the appropriate unit when measuring area
- explains the relationship between the length, breadth and area of squares and rectangles
- explains the relationship between the base, perpendicular height and area of triangles

Related syllabus content
on p 99

## Stage 4

## Perimeter and Area <br> MS4. 1

Uses formulae and Pythagoras' theorem in calculating perimeter and area of circles and figures composed of rectangles and triangles
The student, for example:

- converts between metric units of length
- finds the perimeter of squares, rectangles and simple composite figures
- makes reasonable estimates of lengths, perimeters and areas in the school environment
- compares the areas of different rooms
- labels the hypotenuse in right-angled triangles presented in any orientation
- calculates the circumference and area of circles given the radius or diameter
- uses Pythagoras' theorem to find lengths of sides in rightangles triangles


## Measurement

## Volume and Capacity

Students recognise the attribute of volume and use informal and metric units for measuring capacity or volume

## Early Stage 1

## MES1. 3

Compares the capacities of containers and the volumes of objects or substances using direct comparison

The student, for example:

- fills and empties a variety of containers using different materials eg water, sand, marbles, blocks
- recognises when a container is full, empty and about half full
- explains that one container 'has more' or 'has less' capacity than another container
- explains that one container 'will hold more', 'will hold less' or 'will hold about the same' as another container
- compares capacities by pouring materials from one container into another
- compares capacities by packing materials from one container into another
- compares the volumes of two piles of materials by filling two identical containers
- describes the amount of space occupied by objects eg 'The garbage truck takes up more space than a car.'

Related syllabus content on p 102

## Stage 1

## MS1. 3

Estimates, measures, compares and records volumes and capacities using informal units

The student, for example:

- counts and compares the number of cups of sand or water needed to fill two different containers
- recognises that two containers of different shape may hold the same amount of material eg 'This short fat cup holds about the same amount of drink as this tall thin glass.'
- estimates and measures the capacity of a container using informal units
- orders three containers according to their capacity
- calibrates a clear bottle using a cup as the informal unit
- selects an appropriate informal unit to measure and compare the capacities of two containers
- compares the capacities of two containers by filling each and counting the number of informal units used
- builds models using blocks and compares their volume by counting the number of identical blocks used
- orders three models according to their volume
- compares the volumes of two objects by marking the change in water level when each is submerged

Related syllabus content
on p 103

## Measurement

## Volume and Capacity

Students recognise the attribute of volume and use informal and metric units for measuring capacity or volume

## Stage 2

## MS2. 3

Estimates, measures, compares and records volumes and capacities using litres, millilitres and cubic centimetres

The student, for example:

- selects from a range of containers those that have a capacity of more than, less than and about one litre
- uses the abbreviation for litre (L) and millilitre (mL)
- estimates and measures the capacity of containers to the nearest litre
- gives examples of situations where a unit smaller than the litre is needed for measurement
- estimates, measures and compares volume and capacity using millilitres
- describes the litre as being the same as 1000 millilitres
- compares packaging quantities measured in millilitres
- compares the volumes of three objects by marking the change in water level when each is submerged in a container
- measures volume using cubic centimetres

Related syllabus content
on pp 104-105

## Stage 3

## MS3. 3

Selects and uses the appropriate unit to estimate and measure volume and capacity, including the volume of rectangular prisms

The student, for example:

- estimates then measures the volume of a rectangular prism built from cubic centimetre blocks by counting the blocks
- estimates then measures the capacity of a rectangular container using centimetre blocks
- identifies instances where capacity is measured in cubic metres
- explains the relationship between the length, breadth, height and volume of rectangular prisms
- recognises the relationship between one millilitre and one cubic centimetre
- selects a cube with a volume of one cubic centimetre from a collection of other cubes
- calculates the volume of an irregular solid by submerging it in water and measuring the water displaced

Related syllabus content
on p 106

## Stage 4

## Surface Area and Volume MS4.2

Calculates surface area of rectangular and triangular prisms and volume of right prisms and cylinders
The student, for example:

- calculates the surface area of rectangular and triangular prisms
- calculates the volume of right prisms and cylinders
- measures the dimensions of an object and calculates its volume
- measures and calculates the surface area of a package that is a rectangular prism
- draws two containers with the same volume but different dimensions
- calculates the capacity of containers that are in the shape of prisms and cylinders

Related syllabus content
on p 107

## Measurement

## Mass

Students recognise the attribute of mass through indirect and direct comparisons, and use informal and metric units for measurement

## Early Stage 1 <br> Stage 1

## MES1.4

Compares the masses of two objects and describes mass using everyday language

The student, for example:

- describes objects in terms of their mass using everyday language eg heavy, light, hard to lift
- describes the mass of an object as being 'heavier' or 'lighter' than another object
- describes which object is harder to push or pull eg 'The big block was harder to push than the crayon.'
- determines which of two objects is heavier or lighter by hefting
- sorts objects into light and heavy groups
- discusses the action of an equal arm balance when a heavy object is placed in one pan and a lighter object in the other
- determines which of two objects is heavier or lighter by using an equal arm balance

Related syllabus content on p 108

## MS1. 4

Estimates, measures, compares and records the masses of two or more objects using informal units

The student, for example:

- uses an equal arm balance to find two objects which have the same mass
- orders the mass of two or more objects by hefting
- estimates, measures and records the mass of an object using informal units and an equal arm balance
- compares and orders the mass of three objects using informal units
- records the mass of an object by referring to the number and type of informal units used eg 'Fifteen teddy bears balanced the book.'
- uses an equal arm balance to find two collections of objects that have the same mass eg a collection of three blocks is the same as a collection of ten counters
- selects an identical informal unit to compare masses
- selects an appropriate informal unit to measure the mass of an object and justifies the choice


## Related syllabus content

 on p 109
## Mathematics K-6

## Measurement

## Mass

Students recognise the attribute of mass through indirect and direct comparisons, and use informal and metric units for measurement

## Stage 2

## MS2.4

Estimates, measures, compares and records masses using kilograms and grams

The student, for example:

- identifies objects that have a mass more than, less than or about the same as one kilogram
- estimates, measures and records the mass of objects to the nearest kilogram or gram using an equal arm balance
- uses the abbreviation for kilograms (kg) and grams (g)
- explains the need for a unit smaller than a kilogram to measure mass
- measures mass using a given measuring device eg a kitchen scale
- converts between kilograms and grams
- estimates and checks the number of similar objects which have a total mass of one kilogram
- orders commercial products by interpreting labelling eg a 1.25 kg box of cereal has a greater mass than a 625 g tin of fruit
- records mass using decimal notation to two decimal places eg 1.25 kg

Related syllabus content on p 110

## Stage 3

## MS3. 4

Selects and uses the appropriate unit and measuring device to find the mass of objects

The student, for example:

- chooses appropriate units to solve problems involving mass
- names objects and materials whose mass is measured in tonnes eg sand, soil, vehicles
- uses the abbreviation for tonne ( t )
- converts between kilograms and tonnes
- selects the appropriate device to measure mass
- selects and uses the appropriate unit to measure mass
- uses decimal notation to three decimal places when recording mass
- relates the mass of one litre of water to one kilogram

Related syllabus content on p 111

## Measurement

## Time

Students develop an understanding of the passage of time, its measurement and representations, through the use of everyday language and experiences

## Early Stage 1

## MES1.5

Sequences events and uses everyday language to describe the duration of activities

The student, for example:

- describes 'daytime' and 'night-time'
- uses the terms 'yesterday', 'today' , 'tomorrow' and 'before' and 'after'
- sorts picture cards into events that happen in the morning, afternoon or night-time
- names and orders the days of the week and identifies week-days and weekend days
- relates an event to a particular day eg 'We have music on Monday.'
- names the seasons
- compares and discusses the duration of two events eg 'It takes me longer to eat my lunch than it does to clean my teeth.'
- reads hour time on analog and digital clocks
- uses the term 'o'clock'

Related syllabus content on p 112

## Stage 1

## MS1.5

Compares the duration of events using informal methods and reads clocks on the half-hour

The student, for example:

- measures the duration of events using informal units
- orders two or more events measured using a repeated informal unit
- names and orders the months of the year
- recalls the number of days that there are in each month
- matches the months of the year to the seasons
- uses a calendar to identify a particular day or date
- uses the terms 'hour', 'minute' and 'second' to describe time
- reads half-hour time on analog and digital clocks
- associates everyday events with particular hour or halfhour times
- indicates when it is thought that an activity has gone for one minute

Related syllabus content on p 113

## Measurement

## Time

Students develop an understanding of the passage of time, its measurement and representations, through the use of everyday language and experiences

## Stage 2

## MS2.5

Reads and records time in one-minute intervals and makes comparisons between time units

The student, for example:

- reads time using the terms 'quarter-past' and 'quarter-to'
- identifies which hour has just passed when the hour hand is not pointing to a numeral
- reads analog and digital clocks to the minute
- relates analog notation to digital notation eg ten to nine is the same as 8:50
- converts between units of time
- reads and interprets simple timetables, timelines and calendars

Related syllabus content on p 114

## Stage 3

## MS3.5

Uses twenty-four hour time and am and pm notation in real-life situations and constructs timelines

The student, for example:

- uses am and pm notation
- uses 24 -hour time notation to tell the time
- converts between 24 -hour notation and am/pm notation
- determines the duration of an event using starting and finishing times
- uses a stop watch to measure the duration of events
- compares local time to the time in another time zone in Australia
- reads timetables from real-life situations involving 24 -hour time
- determines a suitable scale and uses the scale to draw a timeline
- interprets a given timeline using the scale

> Related syllabus content on p 115

## Stage 4

## Time

MS4. 3
Performs calculations of time that involve mixed units

The student, for example:

- calculates differences in time using a calculator
- mentally adds measurements of time
- uses timetables to solve problems
- solves simple problems involving time zones
- plans a journey which satisfies a set of time constraints

Related syllabus content
on p 116

## Space and Geometry

## Three-dimensional Space

Students develop verbal, visual and mental representations of three-dimensional objects, their parts and properties, and different orientations

## Early Stage 1

## SGES1.1

Manipulates, sorts and represents three-dimensional objects and describes them using everyday language

The student, for example:

- describes three-dimensional objects using everyday language eg 'The block of wood is box-shaped.'
- describes the features of three-dimensional objects using everyday language eg flat, round, curved
- sorts three-dimensional objects and explains the attribute used eg colour, size, shape, function
- predicts and describes the movement of an object eg 'This will roll because it is round.'
- makes models from a variety of materials and describes them using everyday language

Related syllabus content on p 118

## Stage 1

## SGS1.1

Models, sorts, describes and represents three-dimensional objects including cones, cubes, cylinders, spheres and prisms, and recognises them in pictures and the environment

The student, for example:

- describes cones, cubes, cylinders, spheres and prisms
- identifies and names cones, cubes, cylinders, spheres and prisms from a collection of everyday objects
- recognises three-dimensional objects in the environment
- matches a photograph or drawing of an object with the actual object
- uses the terms 'faces', 'edges' and 'corners' to describe three-dimensional objects
- sorts three-dimensional objects according to a particular attribute eg shape of faces
- recognises that three-dimensional objects look different from different views

Related syllabus content on p 119

## Space and Geometry

## Three-dimensional Space

Students develop verbal, visual and mental representations of three-dimensional objects, their parts and properties, and different orientations

## Stage 2

## SGS2.1

Makes, compares, describes and names three-dimensional objects including pyramids, and represents them in drawings

The student, for example:

- describes the features of prisms, pyramids, cylinders, cones and spheres
- identifies and names groups of three-dimensional objects as prisms, pyramids, cylinders, cones and spheres
- identifies prisms, pyramids, cylinders, cones and spheres from descriptions
- makes models of three-dimensional objects given a picture or photograph to view
- makes skeletal models of three-dimensional objects
- sketches a three-dimensional model, attempting to show depth
- sketches three-dimensional objects from different views including top, front and side views
- recognises that prisms have a uniform cross-section

Related syllabus content
on p 120

## Stage 3

## SGS3. 1

Identifies three-dimensional objects, including particular prisms and pyramids, on the basis of their properties, and visualises, sketches and constructs them given drawings of different views

The student, for example:

- describes similarities and differences between different pyramids
- names prisms and pyramids according to the shape of their base
- describes and lists some of the properties of threedimensional objects
- constructs a model of a three-dimensional object given an isometric drawing
- visualises and sketches a three-dimensional object from different views
- visualises and sketches a variety of nets for a given threedimensional object
- draws three-dimensional objects showing simple perspective

Related syllabus content on p 121

## Stage 4

## Properties of Solids <br> SGS4. 1

Describes and sketches three-dimensional solids including polyhedra, and classifies them in terms of their properties

The student, for example:

- describes prisms, cylinders, pyramids, cones and spheres in terms of their geometric properties
- describes the cross-section of three-dimensional solids
- distinguishes between right pyramids and oblique pyramids
- sketches a model made from cubes on isometric grid paper
- counts systematically the vertices, faces and edges of a polyhedron

Related syllabus content on pp 122-123

## Space and Geometry

## Two-dimensional Space

Students develop verbal, visual and mental representations of lines, angles and two-dimensional shapes, their parts and properties, and different orientations

## Early Stage 1 <br> Stage 1

## SGES1.2

Manipulates, sorts and describes representations of twodimensional shapes using everyday language

The student, for example:

- identifies and draws straight and curved lines
- describes closed shapes and open lines
- manipulates a two-dimensional shape and describes its features using everyday language
- sorts shapes into groups according to size or shape and describes each group
- identifies and names a circle, square, triangle and rectangle presented in different orientations
- identifies shapes in the environment
- makes shapes using a variety of materials
- creates different shapes using a computer drawing program
- turns two-dimensional shapes to fit into a given space

Related syllabus content on p 124

## SGS1. 2

Sorts, represents, describes and explores various twodimensional shapes

The student, for example:

- identifies, describes and records the number of sides and corners of various two-dimensional shapes
- describes features of hexagons, rhombuses and trapeziums
- identifies and sorts two-dimensional shapes by a given attribute eg number of sides
- names hexagons, rhombuses and trapeziums presented in different orientations
- uses drawing and painting to represent two-dimensional shapes
- makes as many different shapes as possible by combining two shapes that are the same
eg using two triangles to make

- draws a single line of symmetry on appropriate shapes
- makes symmetrical designs with pattern blocks, drawings and paintings
- identifies shapes that do and do not tessellate
- identifies and names parallel, vertical and horizontal lines in pictures and the environment
- compares angles by placing one angle on top of another

Related syllabus content on p 125

## Space and Geometry

## Two-dimensional Space

Students develop verbal, visual and mental representations of lines, angles and two-dimensional shapes, their parts and properties, and different orientations

## Stage 2

## SGS2.2a

Manipulates, compares, sketches and names two-dimensional shapes and describes their features

The student, for example:

- identifies pentagons, octagons and parallelograms presented in different orientations
- describes the features of special groups of quadrilaterals
- uses measurement to describe the features of a twodimensional shape eg the opposite sides of a parallelogram are the same length
- groups two-dimensional shapes using multiple attributes eg shapes with parallel sides and right angles
- compares the rigidity of three-sided frames with the rigidity of four-sided frames
- identifies all lines of symmetry for a given shape

> Related syllabus content
> on p 126

## SGS2.2b

Identifies, compares and describes angles in practical situations

The student, for example:

- identifies and names perpendicular lines
- identifies an angle with two arms in practical situations
- identifies angles in two-dimensional shapes and threedimensional objects
- identifies the arm and vertex of the angle in an opening, a slope and a turn where one arm is visible
- compares angles using an angle tester

Related syllabus content on p 127

## Stage 3

## SGS3.2a

Manipulates, classifies and draws two-dimensional shapes and describes side and angle properties

The student, for example:

- compares and describes the properties of isosceles, equilateral and scalene triangles
- draws regular and irregular two-dimensional shapes given a description of their side and angle properties
- uses a ruler, set square, protractor or template to draw regular and irregular two-dimensional shapes
- identifies and names the centre, radius, diameter and circumference of a circle
- identifies and names shapes that have rotational symmetry
- enlarges or reduces a graphic or photograph using a computer program

> Related syllabus content on p 128

## SGS3.2b

Measures, constructs and classifies angles

The student, for example:

- identifies the arms and vertex of an angle where both arms are invisible, such as in rotations and rebounds
- measures and constructs angles in degrees using a protractor
- classifies angles as right, acute, obtuse, reflex, straight or a revolution
- measures angles in a quadrilateral to determine whether it is a rectangle or a parallelogram
- identifies angle types as intersecting lines

> Related syllabus content on p 129

## Space and Geometry

## Stage 4

## Angles

## SGS4. 2

Identifies and names angles formed by the intersection of straight lines, including those related to transversals on sets of parallel lines, and makes use of the relationships between them

The student, for example:

- names angles in a diagram
- finds the complement and supplement of an angle
- finds the size of all angles formed when two lines intersect, given the size of one of the angles
- finds the size of all angles formed when two parallel lines are intersected by a transversal, given the size of one of the angles

Related syllabus content on p 130

## Properties of Geometrical Figures SGS4.3

Classifies, constructs, and determines the properties of triangles and quadrilaterals

The student, for example:

- names special types of triangles and quadrilaterals
- draws and labels a diagram from a set of simple specifications for a given triangle or quadrilateral
- recognises particular triangles and quadrilaterals embedded in composite figures
- lists the properties of specific triangles and quadrilaterals
- applies the angle sum of a triangle result, to find the third angle in a triangle
- finds the fourth angle in a quadrilateral given three of the angles
- solves simple numerical problems related to triangles and quadrilaterals

Related syllabus content on pp 131-132

## Space and Geometry

## Stage 4

## Properties of Geometrical Figures

## SGS4. 4

Identifies congruent and similar two-dimensional figures stating the relevant conditions

The student, for example:

- explains the difference between figures that are congruent and those that are similar
- matches the angles of similar or congruent figures when naming the figures
- draws congruent figures using geometrical instruments
- enlarges or reduces a diagram given a scale factor
- calculates the dimensions of similar figures using the enlargement or reduction factor


## Related syllabus content

on p 133

## Space and Geometry

## Position

Students develop their representation of position through precise language and the use of grids and compass directions

## Early Stage 1

## SGES1.3

Uses everyday language to describe position and give and follow simple directions

The student, for example:

- follows a simple direction to position an object eg 'Put the blue teddy in the circle.'
- participates in movement games involving turning and direction
- moves to a different position and describes their action to others eg 'I skipped to the library and walked back.'
- describes their position in relation to an object eg 'I am under the tree.'
- describes the position of an object in relation to themselves eg 'The table is behind me.'
- describes the position of an object in relation to another object eg 'The book is inside the box.'

Related syllabus content on p 134

## Stage 1

## SGS1. 3

Represents the position of objects using models and drawings and describes using everyday language

The student, for example:

- makes a simple model of the playground or classroom and describes the position of objects
- follows oral instructions to position objects in models and drawings
- describes the position of an object in a model, photograph or drawing
- uses 'left' or 'right' to describe the position of objects in relation to themselves
- describes the path from one location to another on a drawing
- creates a path using drawing tools on a computer

Related syllabus content on p 135

## Mathematics K-6

## Space and Geometry

## Position

Students develop their representation of position through precise language and the use of grids and compass directions

## Stage 2

## SGS2.3

Uses simple maps and grids to represent position and follow routes

The student, for example:

- describes the location of an object using more than one descriptor eg 'The book is on the third shelf and second from the left.'
- uses a key or legend to locate a specific object
- describes a route on a simple map
- uses simple coordinates on a grid to describe position eg 'The lion's cage is at B3.'
- plots points at given coordinates to create a picture
- uses a compass to find North
- uses an arrow to represent North on a map
- determines the directions $\mathrm{N}, \mathrm{S}, \mathrm{E}$ and W given one of the directions
- uses N, S, E and W to describe the location of an object on a simple map eg 'The treasure is east of the cave.'
- determines the directions NE, NW, SE and SW given one of the directions
- uses NE, NW, SE and SW to describe the location of an object on a simple map eg 'The treasure is north-east of the cave.'

Related syllabus content on p 136

## Stage 3

## SGS3. 3

Uses a variety of mapping skills

The student, for example:

- finds a place on a map given its coordinates
- uses a given map to plan or show a route
- draws and labels a grid on a map
- identifies different scaled representations of the same plan or model
- uses the scale to calculate the distance between two points on a map
- locates a place on a map which is a given direction from a town or landmark eg locates a town that is north-east of Broken Hill
- draws a map from an aerial view


## Related syllabus content

 on p 137

Mathematics K-6 Noos.

## Glossary

This glossary provides brief explanations of the meaning of particular terms within the $\mathrm{K}-6$ syllabus document. It contains those terms that may be new to primary teachers. This is particularly the case for terms that arise in the Stage 4 Content that is included in the $\mathrm{K}-6$ syllabus. The glossary is not intended to address all mathematical terminology used in the document. Terms written in italics have their own alphabetical entry in the glossary.

Arc (of a circle): Part of the circumference of a circle.


## Average: (see Mean)

Capacity: The amount that a container can hold.
Census: Collection of data from a population (eg all Year 5 students) rather than a sample.
Class interval: A subdivision of a set of data eg students' heights may be grouped into class intervals of $150 \mathrm{~cm}-154 \mathrm{~cm}, 155 \mathrm{~cm}-159 \mathrm{~cm}$.
Cluster: A 'crowding' of data round a particular score eg for the set of scores $7,8,19,19,19,20,20,21,21,36$, there is a cluster of scores around the score 20.

Column graph: A graph that uses separated vertical columns or horizontal bars to represent data.
Composite number: A number that has more than two factors
eg 15 is a composite number because it has factors $1,3,5$ and 15 .
Concave quadrilateral: A quadrilateral that contains a reflex angle
eg


Continuous data: Data that can take any value within a given range eg the heights in centimetres of the students in a class.

Conversion graph: A line graph that can be used to convert from one unit to another eg from \$A to \$US.
Cross-section: The shape (plane section) produced when a solid is cut through by a plane, parallel to the base eg the cross-section of a cone is a circle


Cumulative frequency: The total of all frequencies up to and including the frequency for a particular score in a frequency distribution
eg

| Score | Frequency | Cumulative Frequency |
| :---: | :---: | :---: |
| 10 | 6 | 6 |
| 20 | 7 | 13 |
| 30 | 8 | 21 |
| 40 | 4 | 25 |

The cumulative frequency of the score 30 is 21 , since the total of the frequencies up to and including the frequency for 30 is $6+7+8=21$.

Denominator: The lower number of a fraction that represents the number of equal fractional parts a whole has been divided into.

Discrete data: Data that can only take certain values within a given range eg the number of students enrolled in a school.
Divided bar graph: A graph that uses a single bar divided proportionally into sections to represent the parts of a total
eg


Divided Bar Graph of Weekly Expenditure
Dot plot: A data display in which scores are indicated by symbols such as dots or crosses drawn above a horizontal axis eg


Empty number line: An unmarked number line providing a means for students to record their calculation strategies eg jump strategies for addition and subtraction.

> (Unmarked number line)

(Recording of a jump strategy for calculating $46+33$ )

Equilateral triangle: A triangle with all sides equal in length.
Equivalent fractions: Fractions that can be reduced to the same basic fraction ie fractions that have the same value

$$
\text { eg } \frac{1}{2}=\frac{2}{4}=\frac{3}{6}=\frac{4}{8}
$$

Factor: A factor of a given number is a whole number that divides it exactly eg 1, 2, 3, 4, 6 and 12 are the factors of 12 .

Fibonacci numbers: Numbers in the sequence which begins with two ones and in which each subsequent term is given by the sum of the two preceding terms ie the numbers $1,1,2,3,5,8, \ldots$
Figurate numbers: Numbers that can be represented by a geometric pattern of dots eg triangular numbers, square numbers, pentagonal numbers.
Fraction notation: Representation of numbers in the form $\frac{a}{b}$ where $a$ and $b$ are whole numbers and $b$ is not equal to zero.

Frequency distribution (table): A table that lists a set of scores and the frequency of occurrence of each score eg frequency distribution table for the set of scores:

$$
5,5,6,6,6,6,6,7,7,7,7,8,8,8,9
$$

| Score | Frequency |
| :---: | :---: |
| 5 | 2 |
| 6 | 5 |
| 7 | 4 |
| 8 | 3 |
| 9 | 1 |

Frequency histogram: A graph of a frequency distribution that uses vertical columns (with no gaps between them) to represent the frequencies of the individual scores
eg frequency histogram for the data in the example above:


Frequency polygon: A graph of a frequency distribution formed by joining the midpoints of the tops of the columns of a frequency histogram
eg frequency polygon (with histogram) for the data given in the table above:


Hefting: The comparison of objects, holding one in each hand, to determine which is heavier or lighter.

Improper fraction: A fraction in which the numerator is greater than the denominator.
Index (plural: indices): The number expressing the power to which a number or pronumeral is raised eg in the expression $3^{2}$, the index is 2.

Inverse operation: The operation that reverses the effect of the original operation. Addition and subtraction are inverse operations; multiplication and division are inverse operations.

Isosceles triangle: A triangle with two sides equal in length.
Jump strategy: An addition or subtraction strategy in which the student places the first number on an empty number line and then counts forward or backwards firstly by tens and then by ones to perform a calculation. The number of jumps will reduce with increased understanding.
eg $46+33$
Method 1:


Method 2:


Line graph: A graph in which information is represented through plotting and joining points with a line or line segments. Meaning can be attached to the points between the plotted points eg temperature and population trends may be represented using line graphs.

Line symmetry: A figure has line symmetry if one or more lines ('line of symmetry' or 'axis of symmetry') can be drawn that divide the figure into two mirror images.

Linear scale: A scale where equal quantities are represented by equal divisions eg ruler, thermometer.

Mean (or Average): The total of a set of scores divided by the number of scores eg for the scores $4,5,6,6,9,12$, the mean is $\frac{4+5+6+6+9+12}{6}=7$
Median: The middle score when an odd number of scores is arranged in order of size. If there is an even number of scores, the median is the average of the two middle scores eg for the scores $3,3,6,8,9$, the median is 6 ; for the scores $5,6,9,9$, the median is $\frac{6+9}{2}=7 \frac{1}{2}$
Mental facility: The ability to use a variety of strategies to calculate mentally.
Mixed numeral: A number that consists of a whole number part and a fractional part eg $2 \frac{1}{2}$.
Mode: The score that occurs most often in a set of scores ie the score that has the highest frequency. A set of scores may have more than one mode
eg for the scores $1,2,3,3,4,4,4,5$, the mode is 4 ;
for the scores $3,5,5,5,6,6,6,7$, there are two modes, 5 and 6 .
Multiple: A number that is the product of a given number and any whole number greater than zero eg the multiples of 4 are $4,8,12,16,20, \ldots$

Number sense: The ability to use an understanding of number concepts and operations in flexible ways to make mathematical judgements and to develop useful strategies for handling numbers and operations.

Numerator: The upper number of a fraction that represents the number of equal fractional parts.
Oblique prism: (See Prism).
Ogive (or 'cumulative frequency polygon'): A graph formed by joining the top right-hand corners of the columns of a cumulative frequency histogram
eg


Order of rotational symmetry: The number of times a figure coincides with its original position in turning through one full rotation
eg an equilateral triangle has rotational symmetry of order three and a square has rotational symmetry of order four.
Outlier: A score that lies well outside most of the other scores in a set of data
eg 25 is an outlier in the set of scores $1,2,4,4,6,7,25$.
Palindromic numbers: Numbers that are the same if read forward or backwards eg 44, 23532.
Parallelogram: A quadrilateral with both pairs of opposite sides parallel.
Pascal's triangle: A triangular array of numbers bordered by 1's such that the sum of two adjacent numbers is equal to the number between them in the next row.

$$
\begin{aligned}
& 1 \\
& 11 \\
& 121 \\
& 1331 \\
& 14641 \\
& \text { etc }
\end{aligned}
$$

Pentagonal numbers: Numbers that can be represented by a pentagonal pattern of dots. The first five pentagonal numbers 1, 5, 12, 22 and 35 can be represented by


Perimeter: The distance around the boundary of a two-dimensional shape.
Platonic solids: The five regular polyhedra ie the five polyhedra whose faces are regular congruent polygons: tetrahedron (4 faces); cube (6 faces); octahedron (8 faces); dodecahedron (12 faces); icosahedron (20 faces).

The Platonic solids (with nets):

Hexahedron or Cube

Dodecahedron


Icosahedron

Polygon: A two-dimensional shape having three or more straight sides.
Polyhedron (plural: polyhedra): A solid in which each face is a polygon.
Population: The whole group from which a sample is drawn.
Position: The location of an object in relation to oneself or another object.
Prime factor: A prime factor of a given number is a prime number that divides it exactly
eg the prime factors of 42 are 2, 3 and 7.
Prime number: A number that has only two factors, itself and one
eg 3 is a prime number because its only factors are 1 and 3 .
Prism: A solid comprising two congruent parallel faces ('bases') and the ('lateral') faces that connect them.
The lateral faces are parallelograms. If they are all right-angled (ie rectangles) the prism is a 'right prism'; if they are not all right-angled then the prism is an 'oblique prism'
eg


Pyramid: A solid with any polygon as its base. Its other faces are triangles that meet at a common vertex. Pyramids are named according to their base
ega pyramid with a square base is a 'square pyramid'.

Quadrant: A sector with arc equal to a quarter of a circle (and therefore centre angle $90^{\circ}$ ); or (sometimes) an arc equal to a quarter of a circle.


Quantitative data: Data that can be counted (discrete data) or measured (continuous data) eg the number of students enrolled in a school (discrete); the heights in centimetres of the students in a class (continuous).
Range: The difference between the highest and lowest scores in a set of scores eg for the scores $5,7,8,9,10,11$, the range is $11-5=6$.
Rhombus: A parallelogram with all sides equal.
Rhythmic counting: Counting with emphasis on rhythm eg $1,2,3,4,5,6,7,8,9, \ldots$ (where the bold numbers are said more loudly).

Right prism: (see Prism)
Sample: Part of a population chosen so as to give information about the population as a whole.
Scalene triangle: A triangle with no two sides equal in length.
Scatter diagram: A display consisting of plotted points that represent the relationship between two sets of data
eg the scatter diagram shows the Mathematics and English test scores of a class of twenty students. Each point on the diagram represents the pair of scores for one student.


Section: The flat surface obtained by cutting through a solid in any direction
eg the section shown for a square pyramid is a trapezium.


Sector: Part of a circle bounded by two radii and an arc.


Sector (pie) graph: A data display that uses a circle divided proportionally into sectors to represent the parts of a total.
Semicircle: Part (half) of a circle bounded by a diameter and an arc joining the ends of the diameter; or (sometimes) the arc equal to half the circumference of a circle.


Skip counting: Counting forward or backwards in multiples of a particular number eg $3,6,9,12, \ldots$.
Solid: A three-dimensional object.
Square numbers: Numbers that can be represented by a square pattern of dots. The first three square numbers 1, 4, and 9 can be represented by

## .: $: ~$ :

Stem-and-leaf plot: A display that provides simultaneously a rank order of individual scores and the shape of the distribution. The 'stem' is used to group the scores and the 'leaves' indicate the individual scores within each group.

eg $|$| 0 | 569 |
| :--- | :--- | :--- |
| 1 | 1244 |
| 2 | 357 |

(Stem-and-leaf plot for the set of data: $9,6,12,14,14,11,5,23,25,27$.
A back-to-back stem-and-leaf plot has two sets of data displayed on either side of the common stem.
Step graph: A graph that increases or decreases in 'steps' rather than being a continuous line eg


Subitising: The skill of immediately recognising the number of objects in a small collection without having to count the objects.

## Mathematics K-6

Summary statistics: Measures such as mean, mode, median and range used in analysing a set of data.
Translation: Sliding of a figure without rotation or changing of its shape or size.
Trapezium: A quadrilateral with at least one pair of opposite sides parallel.
Travel graph: A graph that represents the relationship between time and distance travelled.
Triangular numbers: Numbers that can be represented by a triangular pattern of dots. The first three triangular numbers 1,3, and 6 can be represented by


Uniform cross-section: A solid has a uniform cross-section if cross-sections taken parallel to its base are always the same size and shape (cross-sections parallel to the base of prisms are uniform, whereas cross-sections parallel to the base of pyramids are not).
Unit fraction: A fraction that has a numerator of one eg $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$
Vertex (plural: vertices): A point where two or more sides of a polygon or edges of a solid meet eg a square has 4 vertices and a cube has 8 vertices.

Visualise: To recreate and manipulate images mentally.
Volume: The amount of space occupied by an object or substance.

